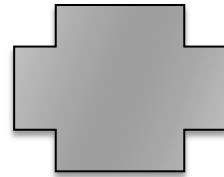


k -Zero-Divisor Hypergraphs

Pinkaew Siriwong

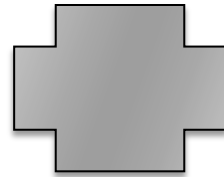
Department of Mathematics and Computer Sciences, Chulalongkorn University

Algebraic
structures



Graph
structures

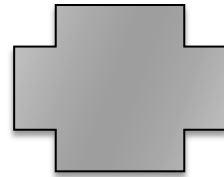
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k-Zero-Divisor

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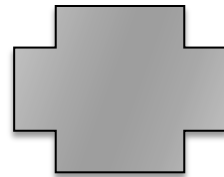


Graph
structures

k-Zero-Divisor

Hypergraph

Algebraic
structures



Graph
structures

k -Zero-Divisor

Hypergraph

k -Zero-divisor hypergraphs

Algebraic structures

Chelvam et.al.



Algebraic structures

A nonzero nonunit element \mathbf{z}_1 is said to be
***k*-zero-divisor**

Algebraic structures

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if there exist $k - 1$ distinct nonunit elements
 $\mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4, \dots, \mathbf{z}_k$ differ from \mathbf{z}_1 such that

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➤ $\mathbf{z}_1 \mathbf{z}_2 \mathbf{z}_3 \cdots \mathbf{z}_k = 0$

➤ the products of elements of any $k - 1$ subset of $\{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \dots, \mathbf{z}_k\}$ are nonzero

Example of k -zero-divisor



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We obtain that $\bar{2}$ is a 3-zero-divisor

Graph structures

Verrall



Graph structures

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- V or $V(\mathcal{H})$ is a nonempty finite set of *vertices* or *vertex set*
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Graph structures

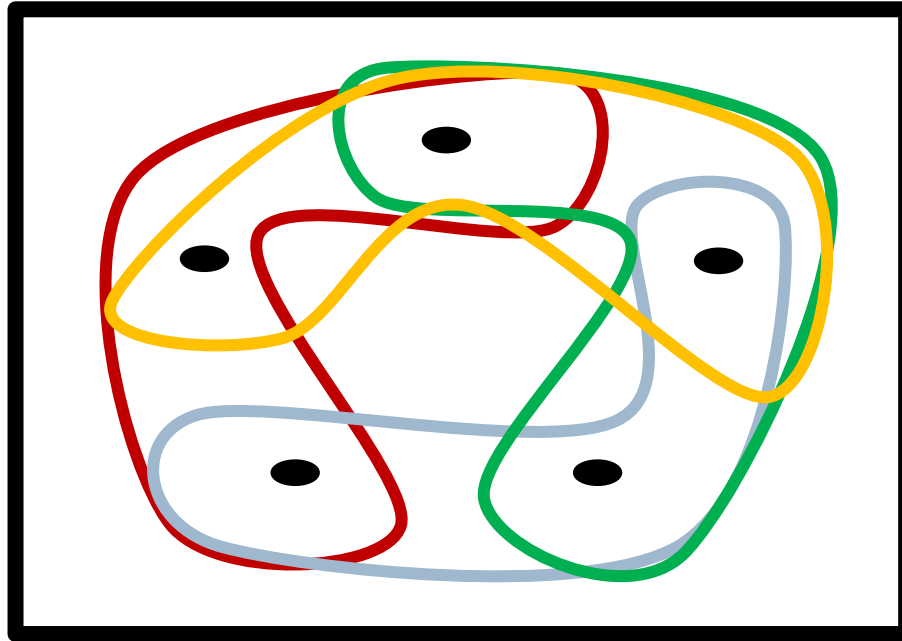
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- If each edge of \mathcal{H} has size l , we call \mathcal{H} an *l -uniform hypergraph*.

Example of hypergraphs



Example of hypergraphs



3-uniform hypergraph

Complete k -uniform hypergraph on n vertices

Verrall

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3

4

Verrall

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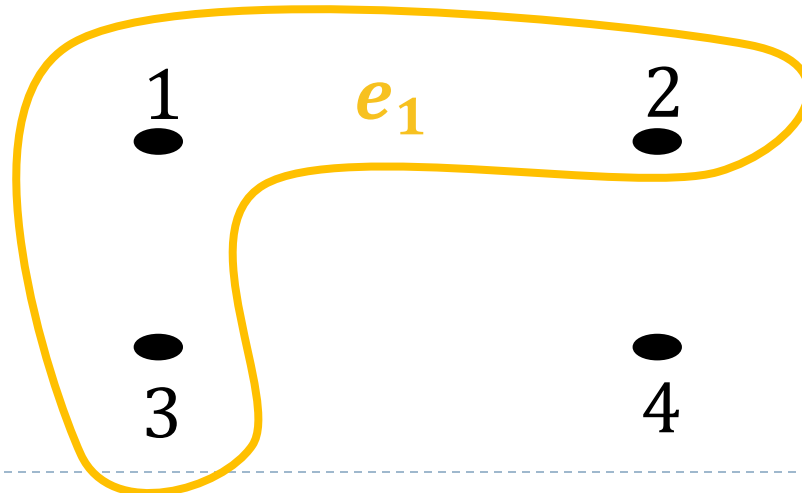
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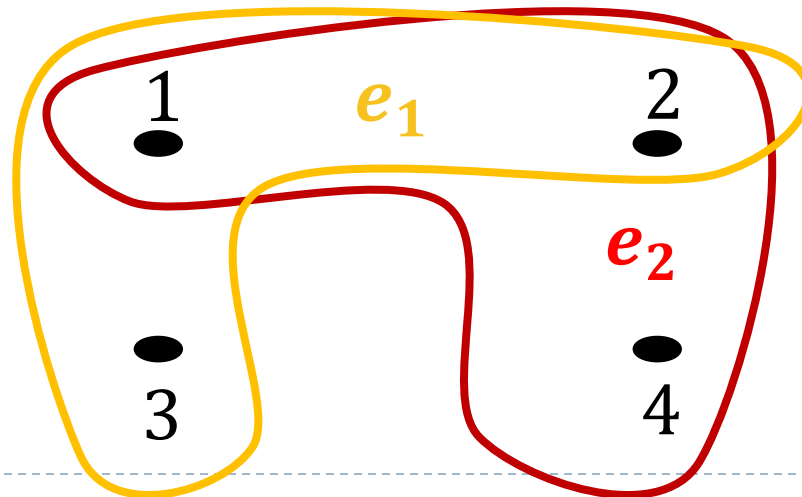
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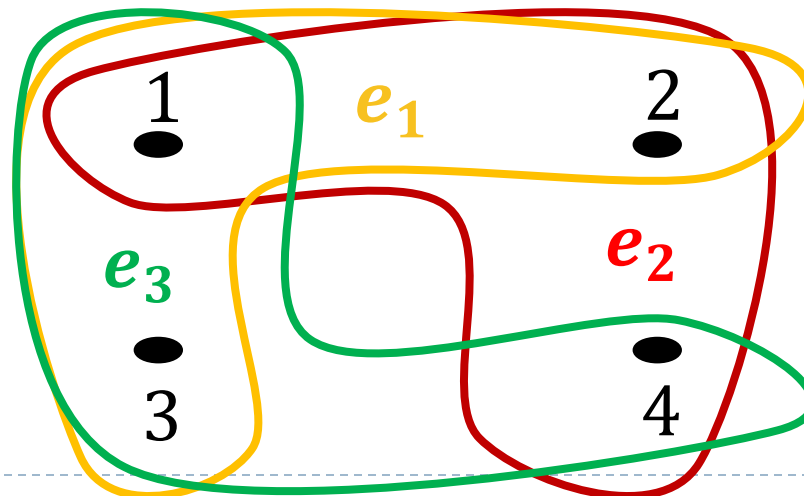
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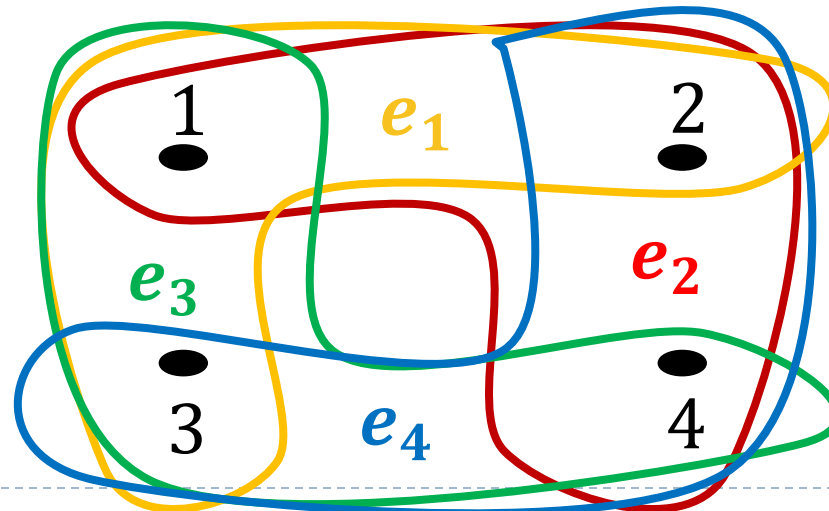
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Verrall



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- Edge set $\mathcal{E} = \{\{v_1, v_2, v_3, \dots, v_k\} \mid v_j \in V_j \text{ for all } 1 \leq j \leq k\}$

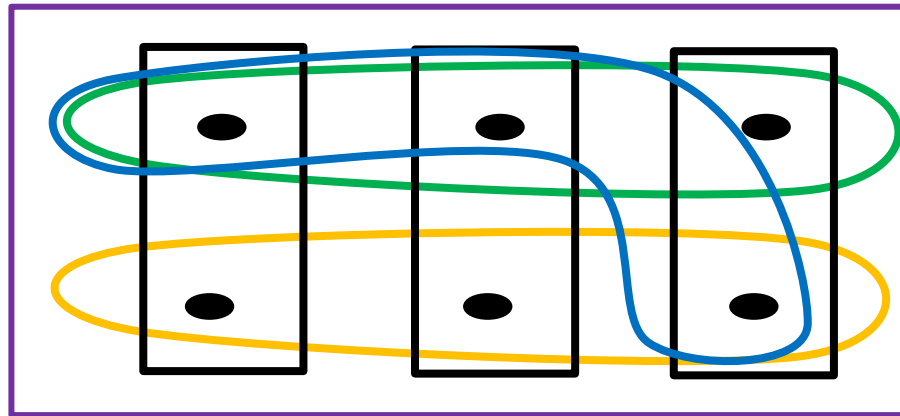
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3-Partite 3-uniform hypergraph

Kuhl and Schroeder

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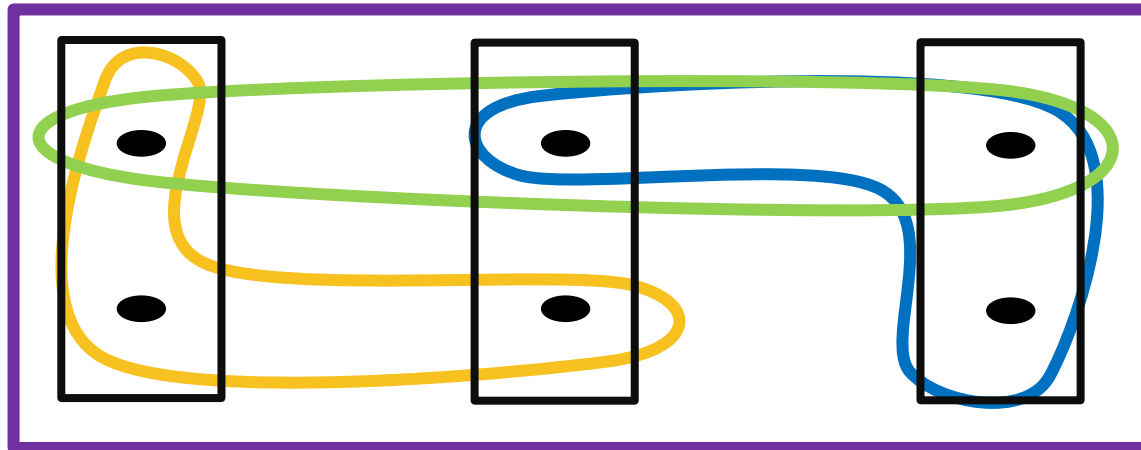
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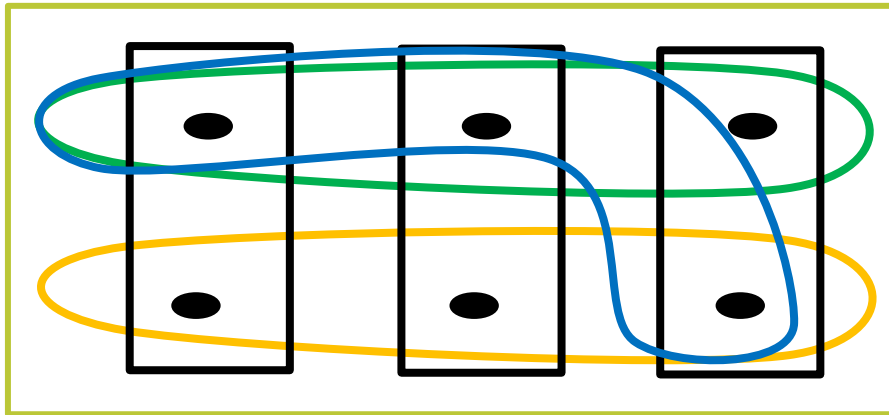


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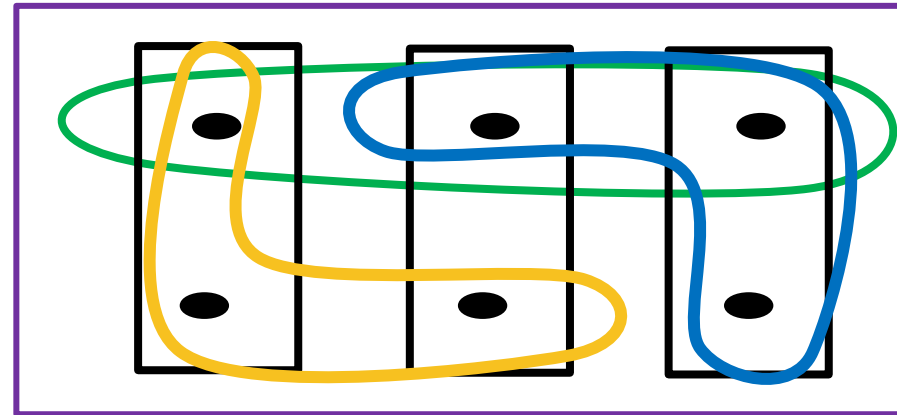
Jirimutu and Wang

3-Partite 3-uniform hypergraph

3-Partite 3-uniform hypergraph



Kuhl and Schroeder



Jirimutu and Wang

Path and Diameter of hypergraph \mathcal{H}

Ye



Path and Diameter of hypergraph \mathcal{H}

➤ A path P from x_1 to x_{s+1} is a vertex-edge alternative sequence $x_1, E_1, x_2, E_2, \dots, x_s, E_s, x_{s+1}$ such that $\{x_i, x_{i+1}\} \subseteq E_i$ for all $1 \leq i \leq s$ and $x_i \neq x_j, E_i \neq E_j$ with $i \neq j$ and s is called the length of the path P .

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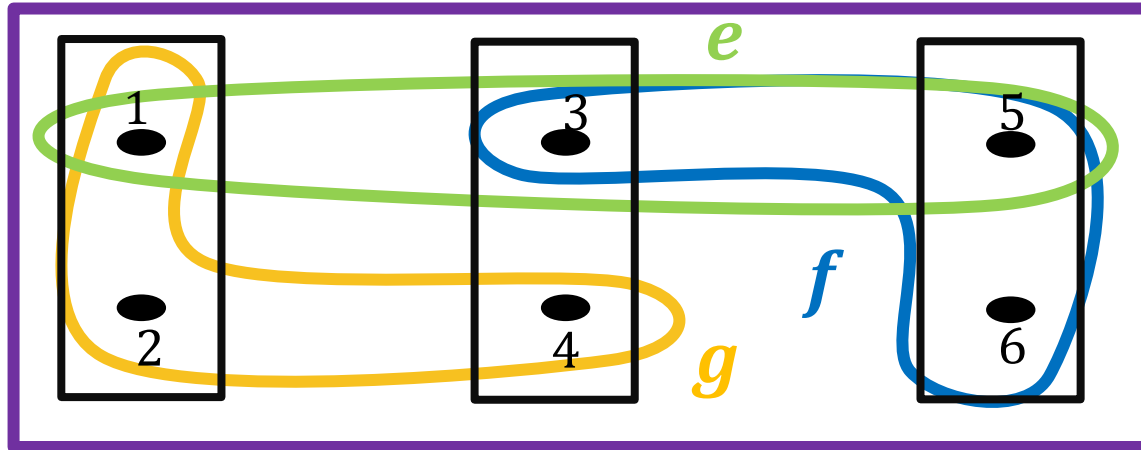
➤ The distance of distinct vertices x and y , denoted by $d(x, y)$, is the minimum length of all paths that connect x and y .

➤ The diameter of $\mathcal{H}(V, \mathcal{E})$, denoted by $d(\mathcal{H})$, is defined as $d(\mathcal{H}) = \max\{d(x, y) | x, y \in V, x \neq y\}$.

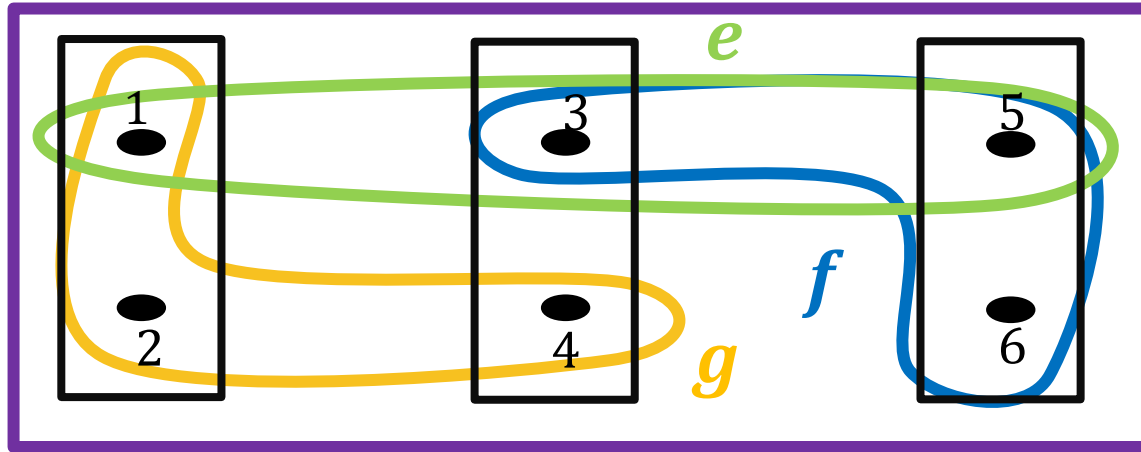


Example of path and diameter

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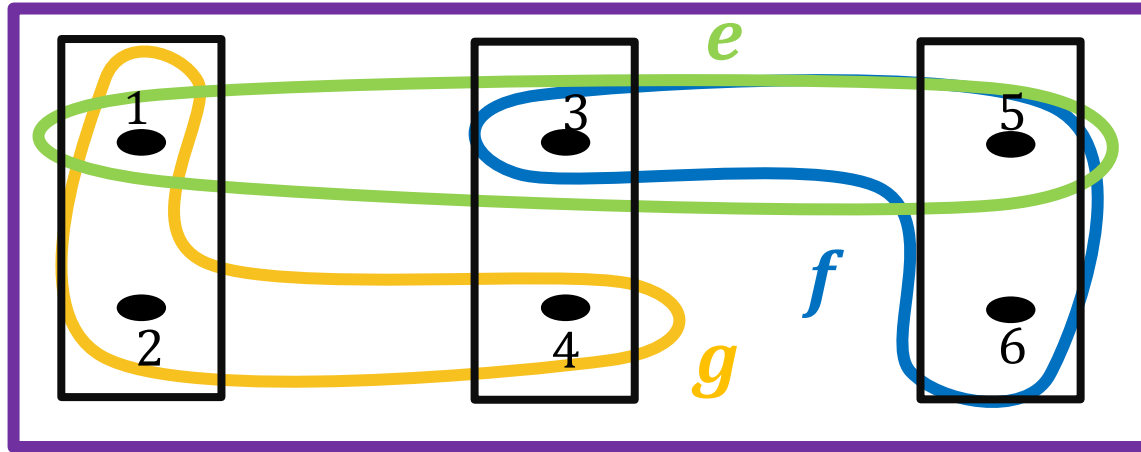


Example of path and diameter



Path from 1 to 6

Example of path and diameter



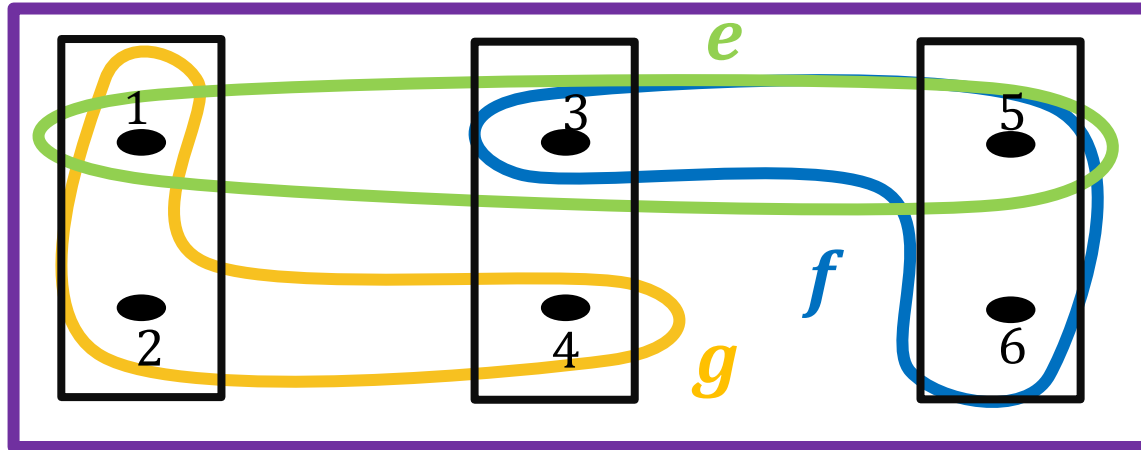
Path from 1 to 6

1, *e*, 3, *f*, 6

1, *e*, 5, *f*, 6

$$d(1,6) = 2$$

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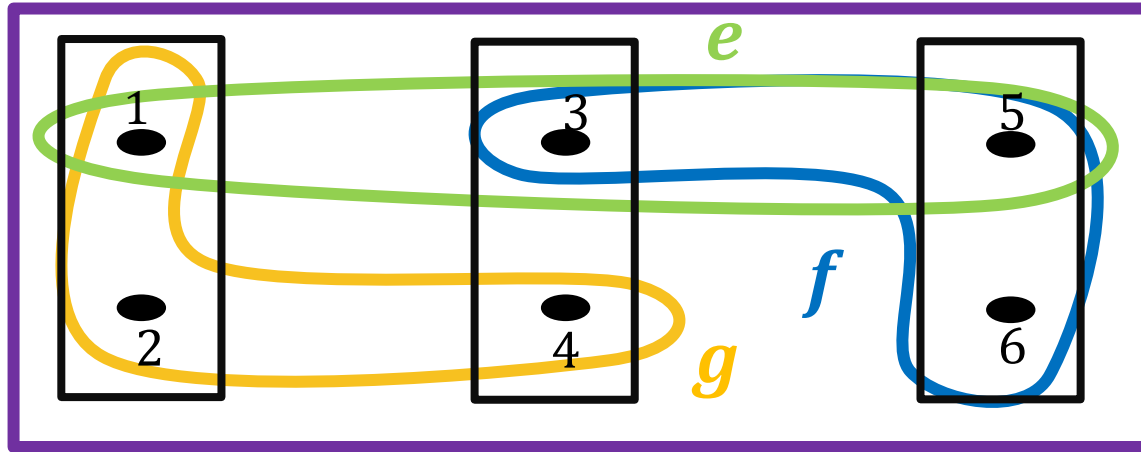
Path from 4 to 6

4, g, 1, e, 3, f, 6

4, g, 1, e, 5, f, 6

$$d(4,6) = 3$$

Example of path and diameter



Path from 1 to 6

1, e, 3, f, 6

1, e, 5, f, 6

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Path from 4 to 6

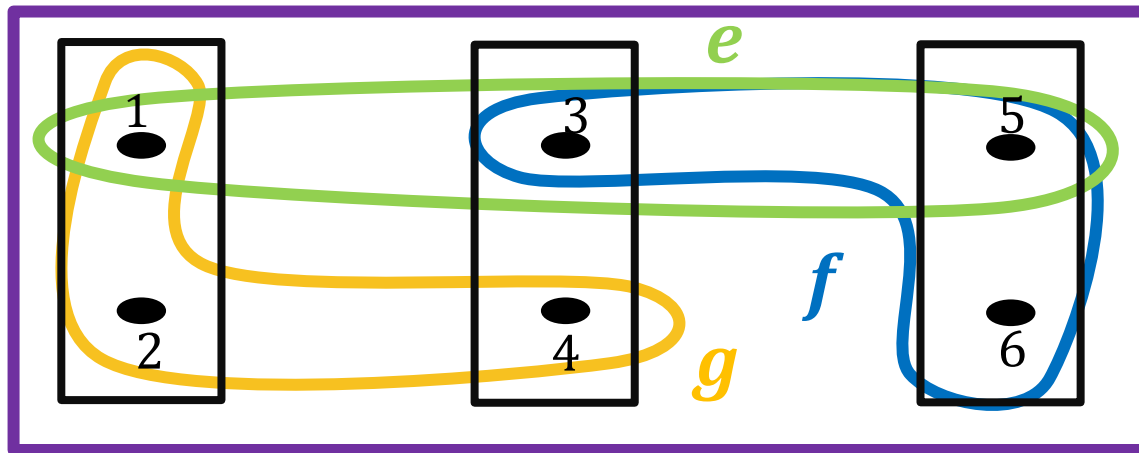
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Consider all $d(x, y)$

Example of path and diameter



Path from 1 to 6

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$$d(\mathcal{H}) = 3$$



Cycle of hypergraph \mathcal{H}

Ye

Cycle of hypergraph \mathcal{H}

► Let $s \geq 2$ be an integer

Ye

Cycle of hypergraph \mathcal{H}

➤ Let $s \geq 2$ be an integer

➤ An s -cycle is an alternating sequence,
 $C = x_1, E_1, x_2, E_2, \dots, x_s, E_s$ of distinct vertices $x_1, x_2, x_3, \dots, x_s$
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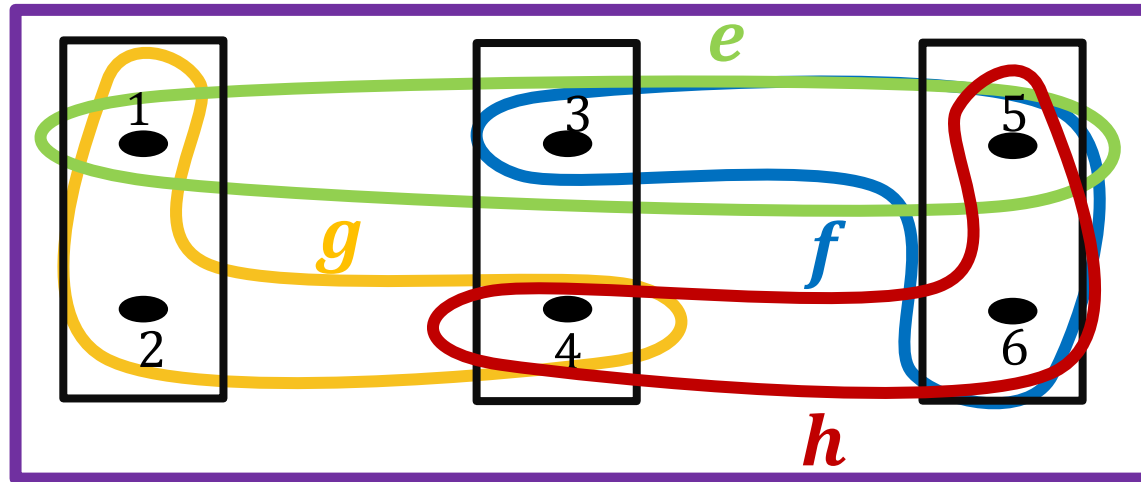
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➤ If hypergraph has no cycle, such hypergraph has 0 -cycle or a
cycle of length 0 .

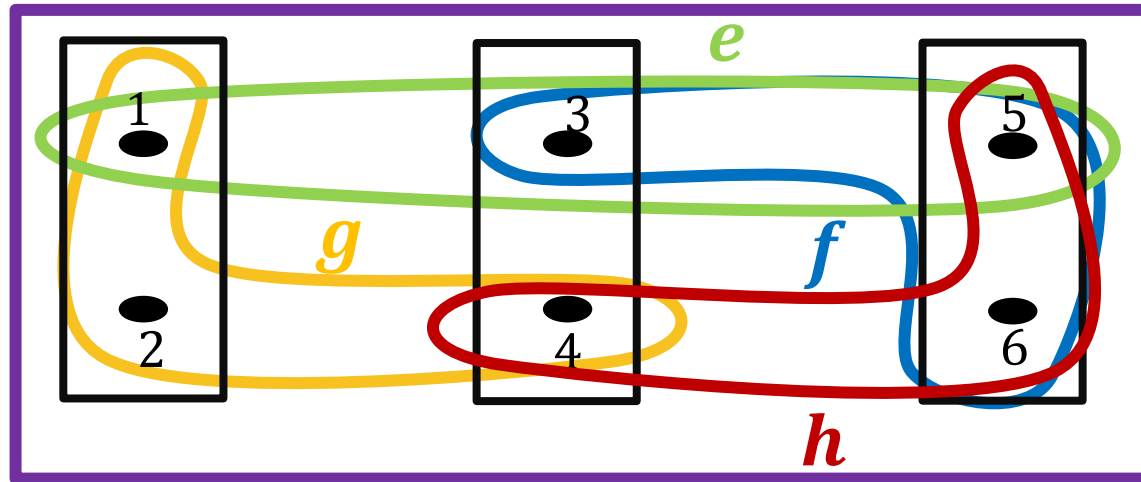
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Example of cycle

Example of cycle

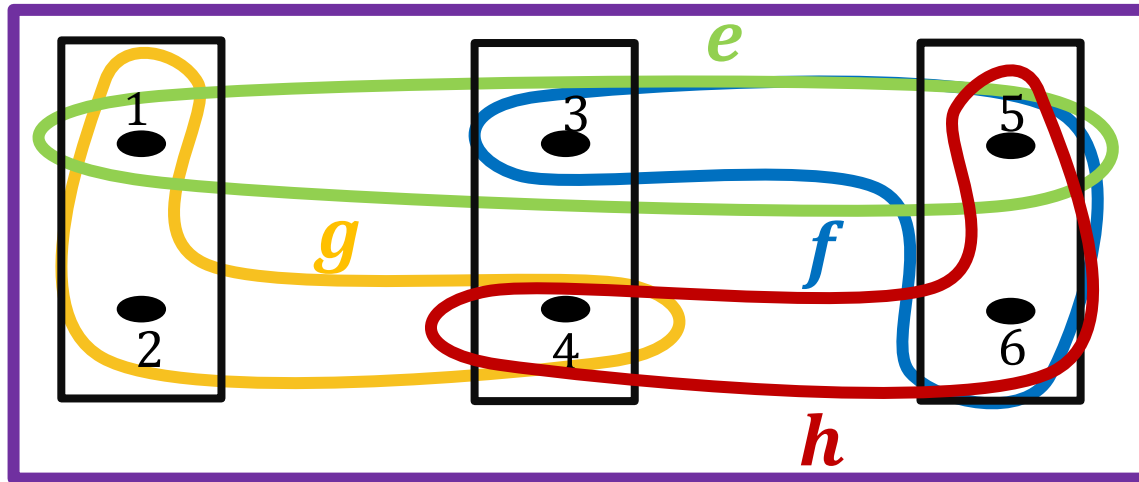


Example of cycle



$$C_1 = 5, e, 3, f$$

Example of cycle

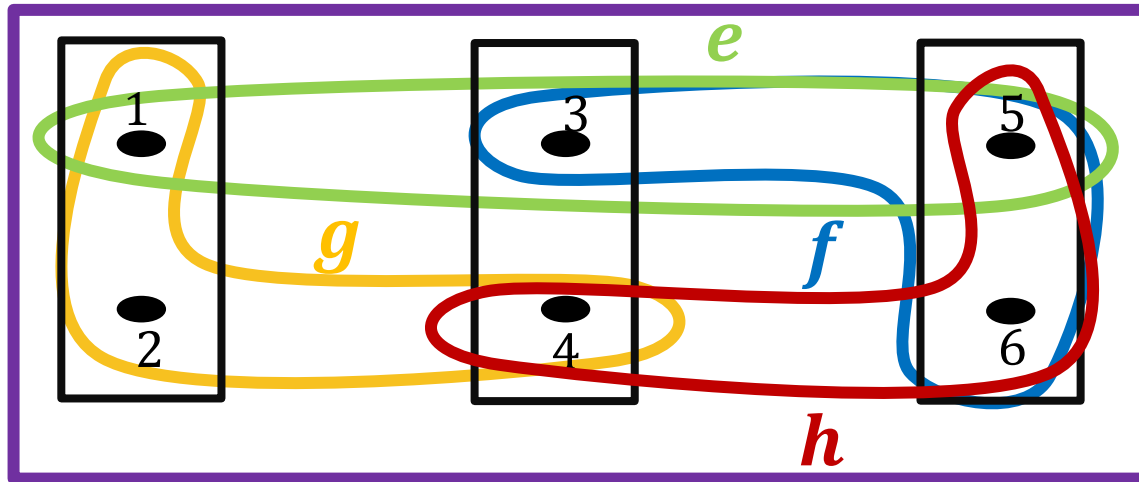


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A cycle of length 2

Example of cycle



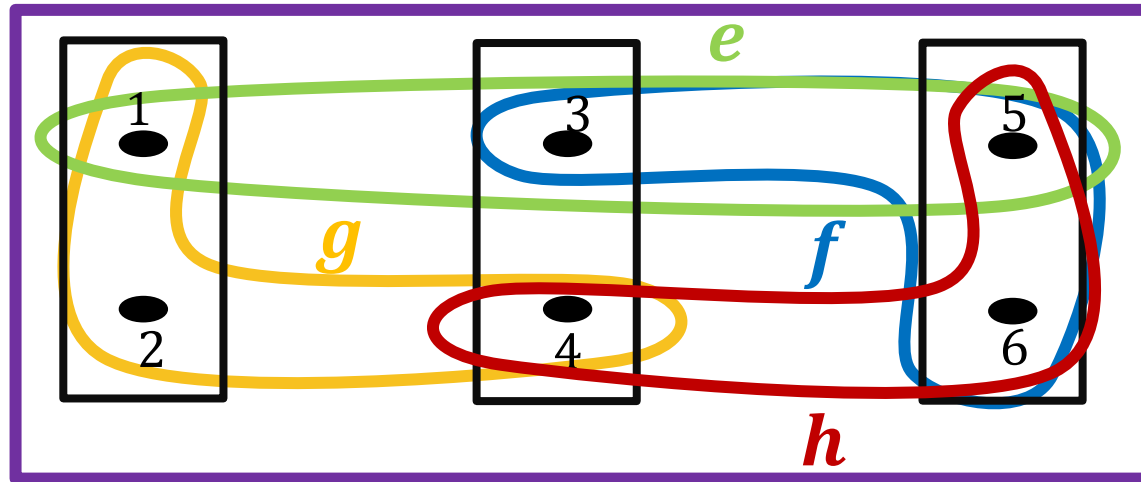
$$C_1 = 5, e, 3, f$$



A cycle of length 2

$$C_2 = 4, h, 6, f, 3, e, 1, g$$

Example of cycle



$$C_1 = 5, e, 3, f$$



A cycle of length 2

$$C_2 = 4, h, 6, f, 3, e, 1, g$$



A cycle of length 4

k -Zero-divisor hypergraphs of a commutative ring R

Chelvam et. al.

k -Zero-divisor hypergraphs of a commutative ring R

➤ $V = Z(R, k)$, set of all k -Zero-Divisors

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➤ We see that k -Zero-Divisor Hypergraphs is k -uniform hypergraph

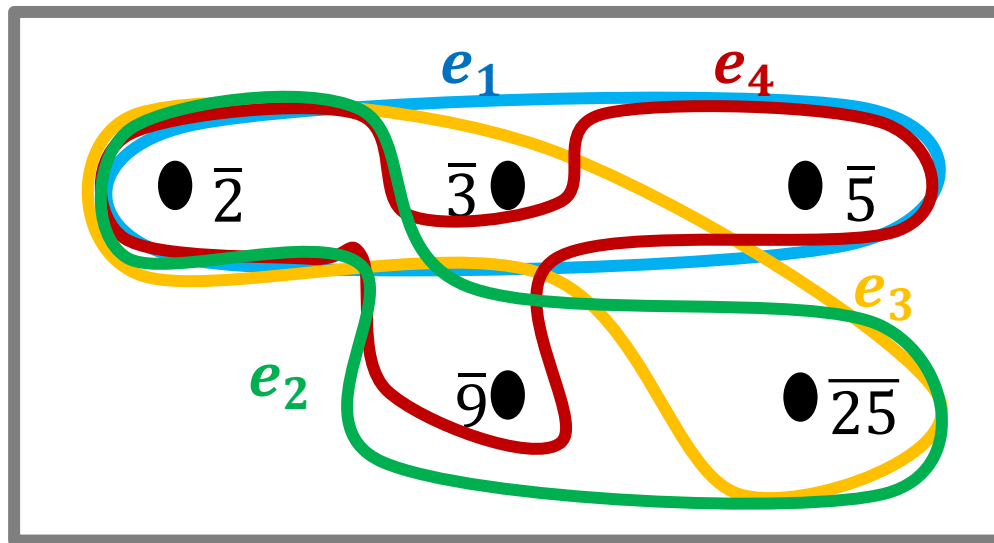
Example of **3**-zero-divisor hypergraphs of \mathbb{Z}_{30}

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$$Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$$

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Complete k -zero-divisor
hypergraph

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hypergraph

Complete k -partite
 k -zero-divisor hypergraph

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k -partite σ -zero-divisor
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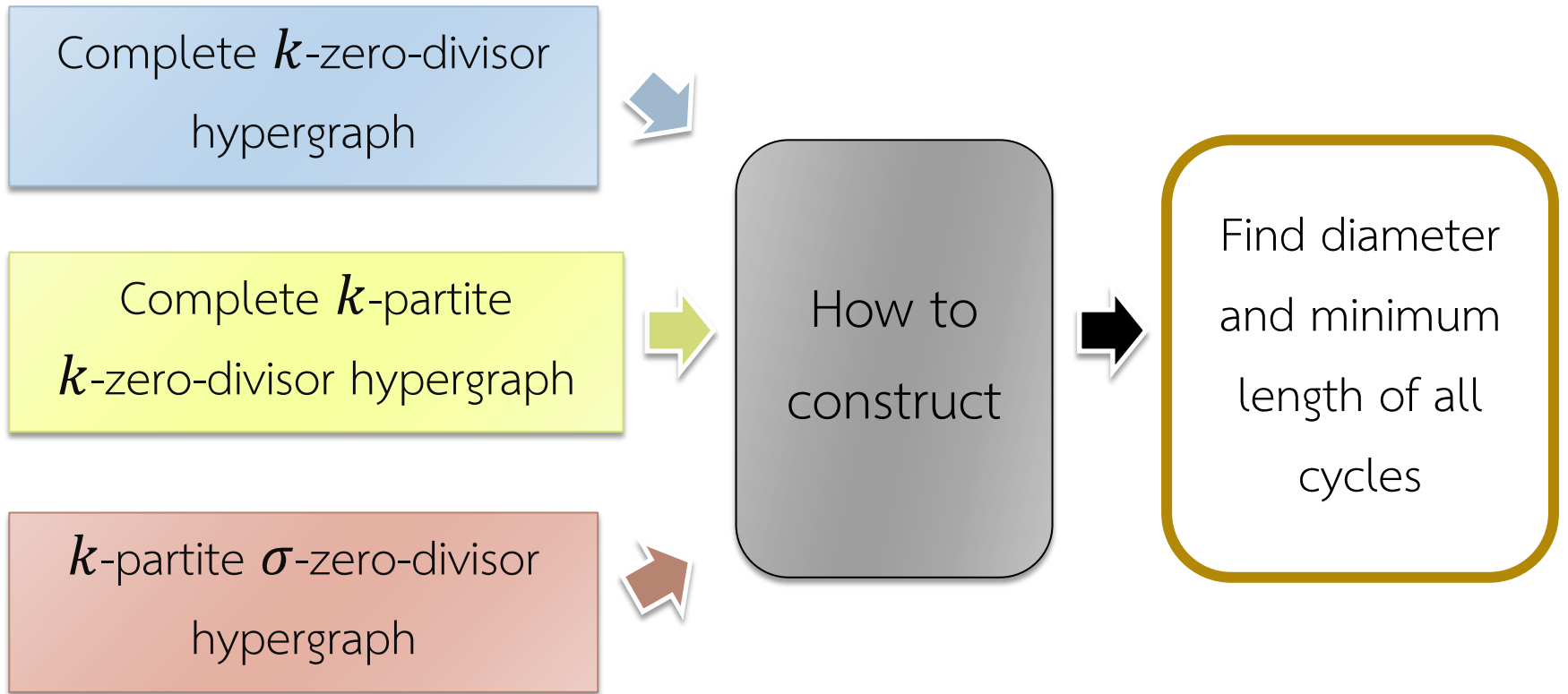
Complete k -partite
 k -zero-divisor hypergraph

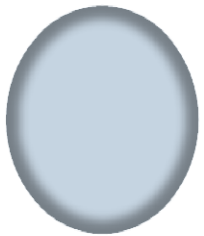
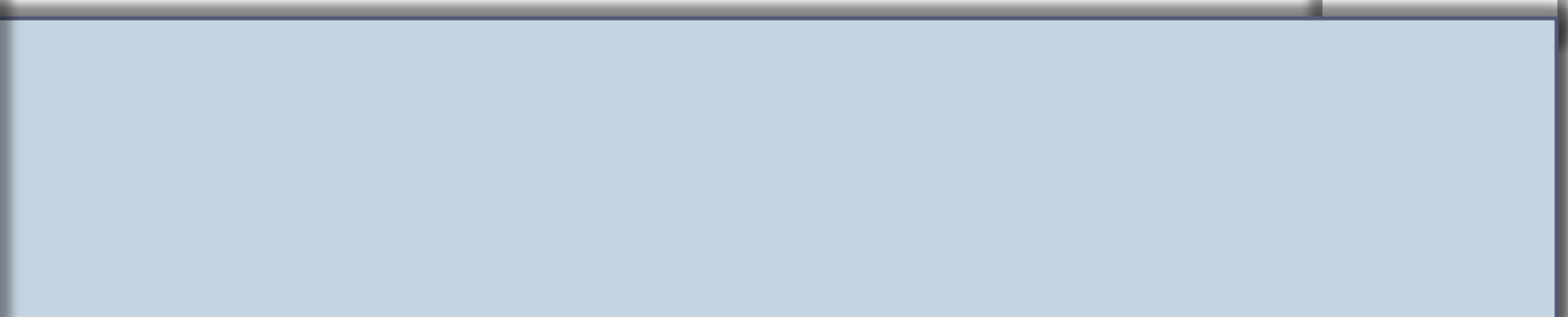
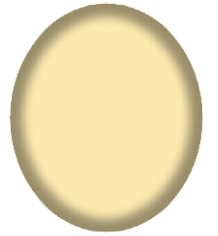
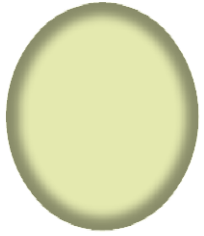


k -partite σ -zero-divisor
hypergraph



How to
construct







Ring R

- Principal ideal domain (PID)
- There exist at least k prime elements $p_1, p_2, p_3, \dots, p_k$



Ring R

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- There exist at least k prime elements $p_1, p_2, p_3, \dots, p_k$

- 
- Finiteness of a ring R/I

Commutative Ring R/I

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Example: $\mathbb{Z}_n \cong \mathbb{Z}/n\mathbb{Z}$

Commutative Ring R/I

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Commutative Ring R/I

Objective

- Find an appropriate ideal so that constructed k -zero-divisor hypergraph has the desired properties



Complete k -zero-divisor hypergraph of ring R



Complete k -zero-divisor hypergraph of ring R

Commutative ring: R/Rp^k

Appropriate ideal: Rp^k

Complete k -zero-divisor hypergraph of ring R

Commutative ring: R/Rp^k

Appropriate ideal: Rp^k

Complete k -zero-divisor hypergraph of ring R

Commutative ring: R/Rp^k

Appropriate ideal: Rp^k



Conditions: $\left| \frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k} \right| \geq k$

Complete k -zero-divisor hypergraph of ring R

Commutative ring: R/Rp^k

Appropriate ideal: Rp^k

Conditions: $\left| Rp/Rp^k - Rp^2/Rp^k \right| \geq k$

Vertex set: $Z\left(R/Rp^k, k\right) = Rp/Rp^k - Rp^2/Rp^k$

Complete k -zero-divisor hypergraph of ring R

Commutative ring: R/Rp^k

Appropriate ideal: Rp^k

Conditions: $\left| \frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k} \right| \geq k$

Vertex set: $Z\left(\frac{R}{Rp^k}, k\right) = \frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k}$

Complete k -zero-divisor hypergraph

Example of complete **3**-zero-divisor hypergraph of ring R



Example of complete **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{27} \cong \mathbb{Z}/27\mathbb{Z} \cong \mathbb{Z}/3^3\mathbb{Z}$

Example of complete **3**-zero-divisor hypergraph of ring R

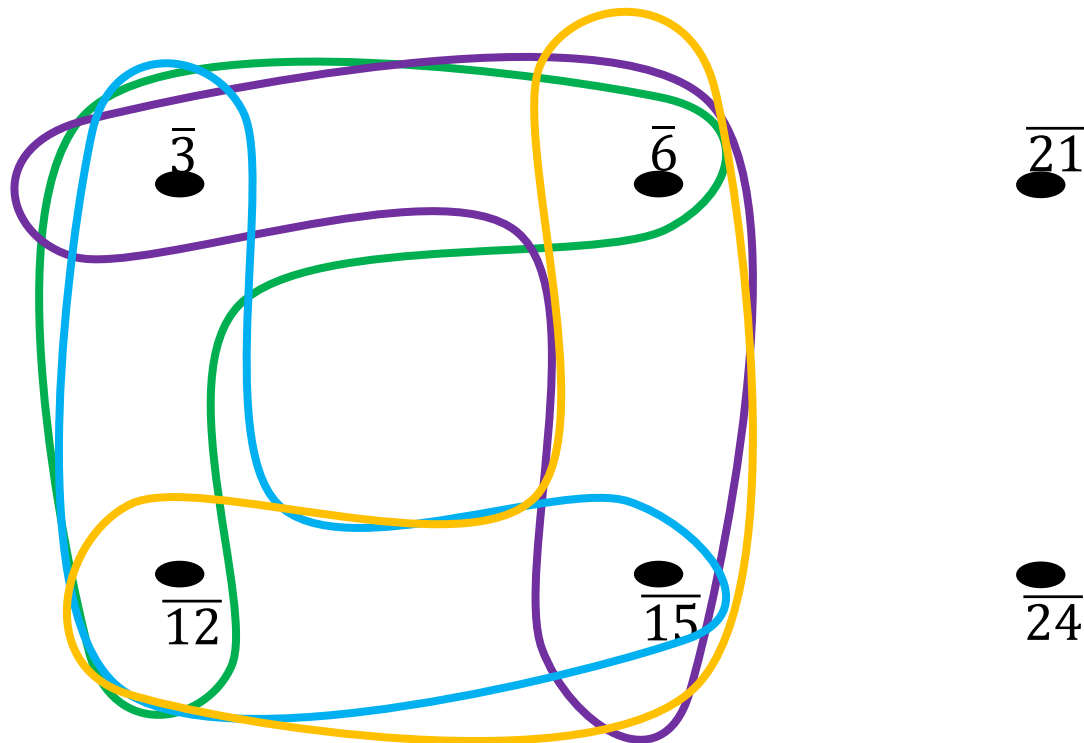
Consider $\mathbb{Z}_{27} \cong \mathbb{Z}/27\mathbb{Z} \cong \mathbb{Z}/3^3\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{27}, 3) = \{\bar{3}, \bar{6}, \bar{12}, \bar{15}, \bar{21}, \bar{24}\}$

Example of complete **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{27} \cong \mathbb{Z}/27\mathbb{Z} \cong \mathbb{Z}/3^3\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{27}, 3) = \{\bar{3}, \bar{6}, \bar{12}, \bar{15}, \bar{21}, \bar{24}\}$



Diameter of complete k -zero-divisor hypergraph of ring R

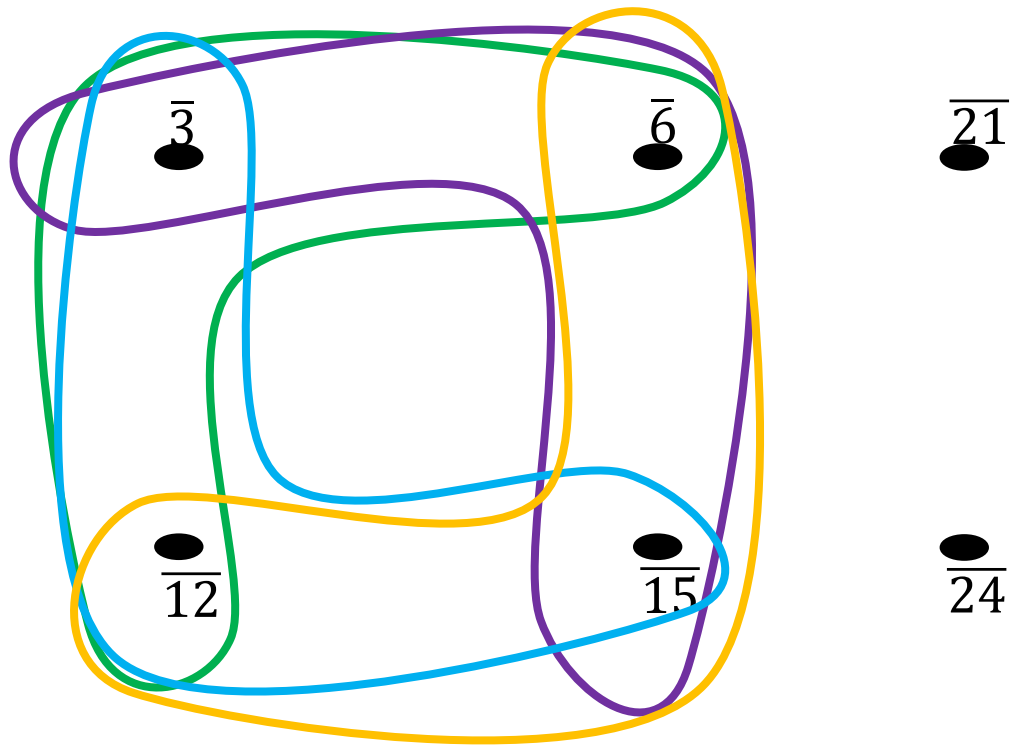


Diameter of complete k -zero-divisor hypergraph of ring R

➤ Diameter is **1** same as a complete graph

Diameter of complete k -zero-divisor hypergraph of ring R

➤ Diameter is 1 same as a complete graph



The minimum length of all cycles

The minimum length of all cycles

$$\triangleright 0 \text{ if } \left| Z \left(R / R p^k, k \right) \right| = k$$

The minimum length of all cycles

➤ 0 if $\left| Z \left(R / Rp^k, k \right) \right| = k$

Only one edge

The minimum length of all cycles

➤ 0 if $\left|Z\left(R/Rp^k, k\right)\right| = k$

Only one edge

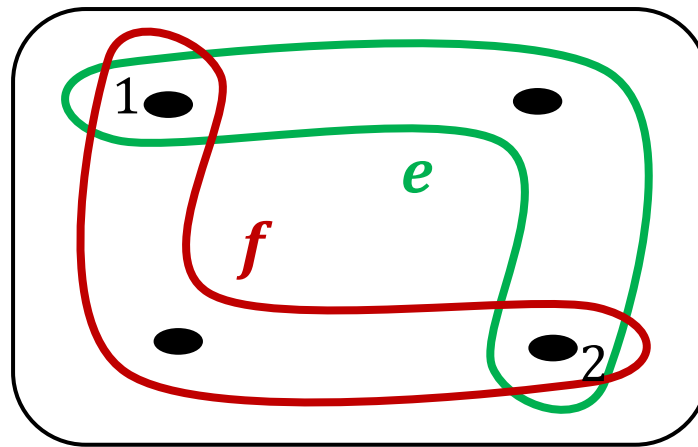
➤ 2 if $k \geq 3$ and $\left|Z\left(R/Rp^k, k\right)\right| \geq k + 1$

The minimum length of all cycles

➤ 0 if $|Z(R/Rp^k, k)| = k$

Only one edge

➤ 2 if $k \geq 3$ and $|Z(R/Rp^k, k)| \geq k + 1$



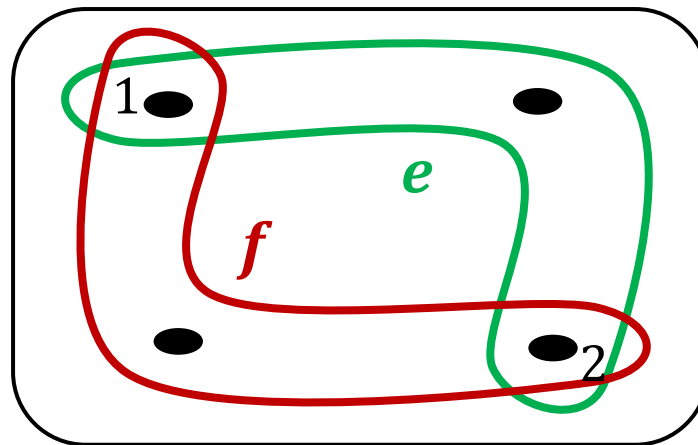
$c = 1, e, 2, f$

The minimum length of all cycles

➤ 0 if $\left|Z\left(R/Rp^k, k\right)\right| = k$

Only one edge

➤ 2 if $k \geq 3$ and $\left|Z\left(R/Rp^k, k\right)\right| \geq k + 1$



$c = 1, e, 2, f$

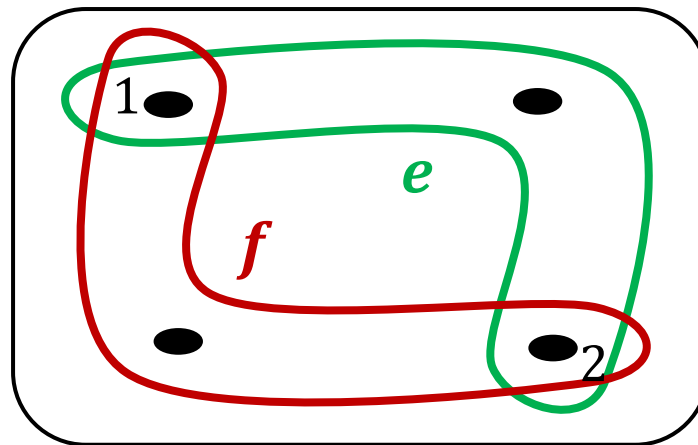
➤ 3 if $k = 2$ and $\left|Z\left(R/Rp^2, 2\right)\right| \geq 3$

The minimum length of all cycles

➤ 0 if $\left|Z\left(R/Rp^k, k\right)\right| = k$

Only one edge

➤ 2 if $k \geq 3$ and $\left|Z\left(R/Rp^k, k\right)\right| \geq k + 1$



$c = 1, e, 2, f$

➤ 3 if $k = 2$ and $\left|Z\left(R/Rp^2, 2\right)\right| \geq 3$

Same idea as complete graph

Conclusion table

Conclusion table

Hypergraph	Appropriate Ideal	Vertex Set	Diameter	Minimum length of all cycles
Complete k -zero-divisor hypergraph	Rp^k	$Z\left(R/Rp^k, k\right) = Rp/Rp^k - Rp^2/Rp^k$	1	0,2, or 3



Complete k -partite k -zero-divisor hypergraph of ring R



Complete k -partite k -zero-divisor hypergraph of ring R

Commutative ring: $R / Rp_1p_2p_3 \cdots p_k$

Appropriate ideal: $Rp_1p_2p_3 \cdots p_k$

Condition: R has at least
 k prime elements

Complete k -partite k -zero-divisor hypergraph of ring R

Commutative ring: $R / Rp_1p_2p_3 \cdots p_k$

Appropriate ideal: $Rp_1p_2p_3 \cdots p_k$

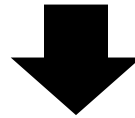
Condition: R has at least
 k prime elements

Complete k -partite k -zero-divisor hypergraph of ring R

Commutative ring: $R / Rp_1p_2p_3 \cdots p_k$

Condition: R has at least
 k prime elements

Appropriate ideal: $Rp_1p_2p_3 \cdots p_k$



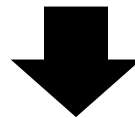
Let $\gamma = p_1p_2p_3 \cdots p_k$

Complete k -partite k -zero-divisor hypergraph of ring R

Commutative ring: $R/Rp_1p_2p_3\cdots p_k$

Condition: R has at least
 k prime elements

Appropriate ideal: $Rp_1p_2p_3\cdots p_k$



Let $\gamma = p_1p_2p_3\cdots p_k$

Each partite set $V_i: R^{p_i}/R\gamma - \bigcup_{j \neq i} R^{p_j}/R\gamma$

$$\bigcup_{i=1}^k V_i = Z^{(R/R\gamma, k)}$$

Complete k -partite k -zero-divisor hypergraph of ring R

Commutative ring: $R/Rp_1p_2p_3 \cdots p_k$

Condition: R has at least k prime elements

Appropriate ideal: $Rp_1p_2p_3 \cdots p_k$

Let $\gamma = p_1p_2p_3 \cdots p_k$

Each partite set $V_i: R^{p_i}/R\gamma - \bigcup_{j \neq i} R^{p_j}/R\gamma$

$$\bigcup_{i=1}^k V_i = Z^{(R/R\gamma, k)}$$

Complete k -partite k -zero-divisor hypergraph



Example of complete **3**-partite **3**-zero-divisor hypergraph of ring R

Example of complete **3**-partite **3**-zero-divisor hypergraph of ring ***R***

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

Example of complete **3**-partite **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$

Example of complete **3**-partite **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$

$\bar{2}$

$\bar{3}$

$\bar{5}$

$\bar{4}$

$\bar{9}$

$\bar{25}$

\vdots

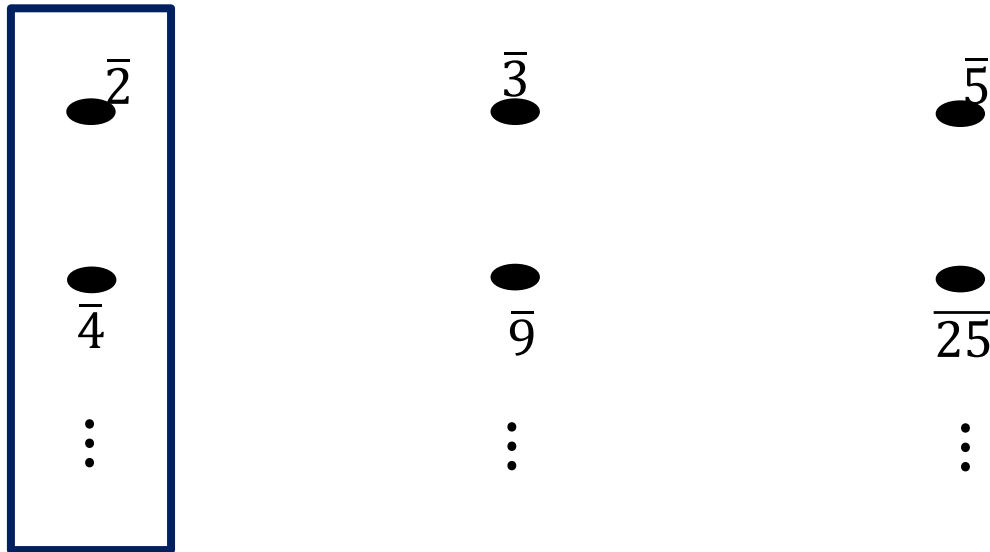
\vdots

\vdots

Example of complete **3**-partite **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$



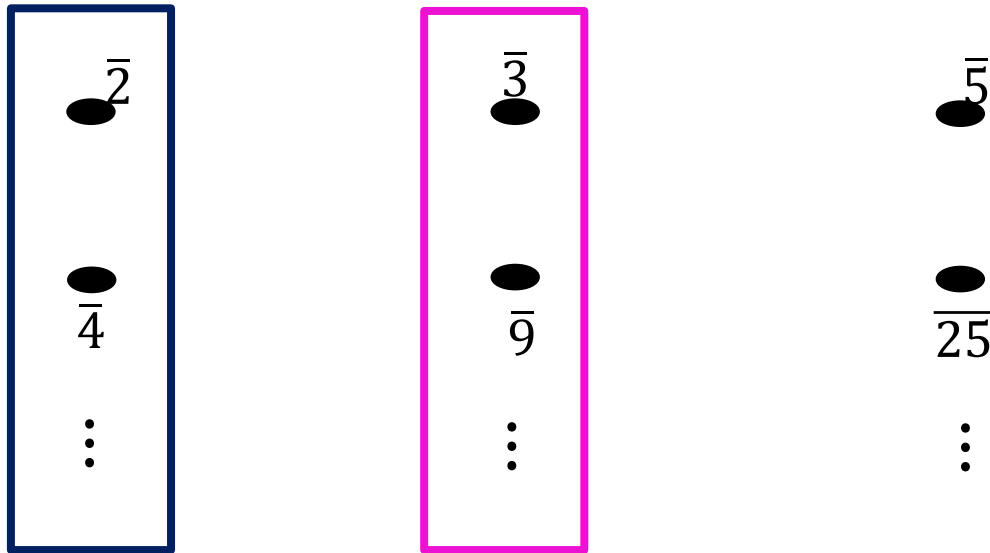
$$V_1 = \{\bar{2}, \bar{4}, \bar{8}, \bar{14}, \bar{16}, \bar{22}, \bar{26}, \bar{28}\}$$



Example of complete **3**-partite **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$



$$V_1 = \{\bar{2}, \bar{4}, \bar{8}, \bar{14}, \bar{16}, \bar{22}, \bar{26}, \bar{28}\}$$

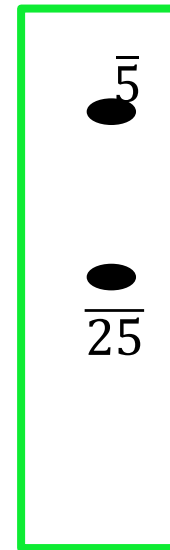
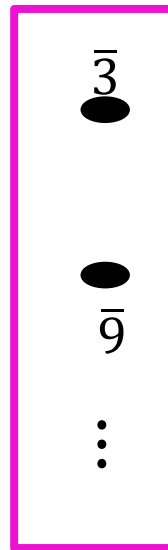
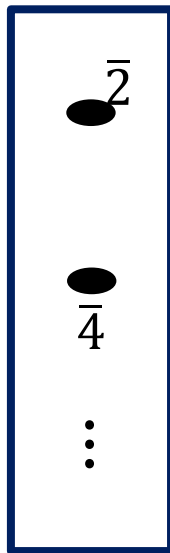
$$V_2 = \{\bar{3}, \bar{9}, \bar{21}, \bar{27}\}$$



Example of complete **3**-partite **3**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$



$$V_3 = \{\bar{5}, \bar{25}\}$$

$$V_1 = \{\bar{2}, \bar{4}, \bar{8}, \bar{14}, \bar{16}, \bar{22}, \bar{26}, \bar{28}\}$$

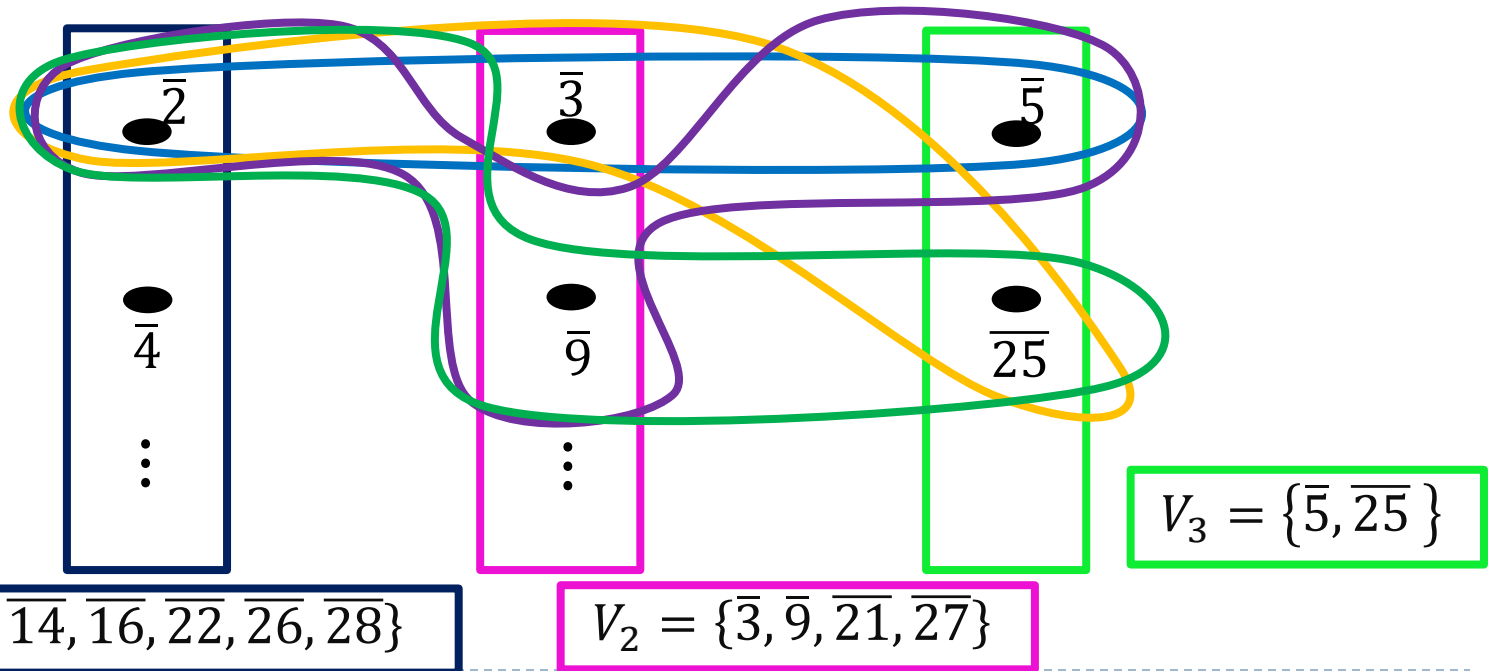
$$V_2 = \{\bar{3}, \bar{9}, \bar{21}, \bar{27}\}$$



Example of complete 3-partite 3-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$



Diameter of complete k -partite k -zero-divisor hypergraph of ring R

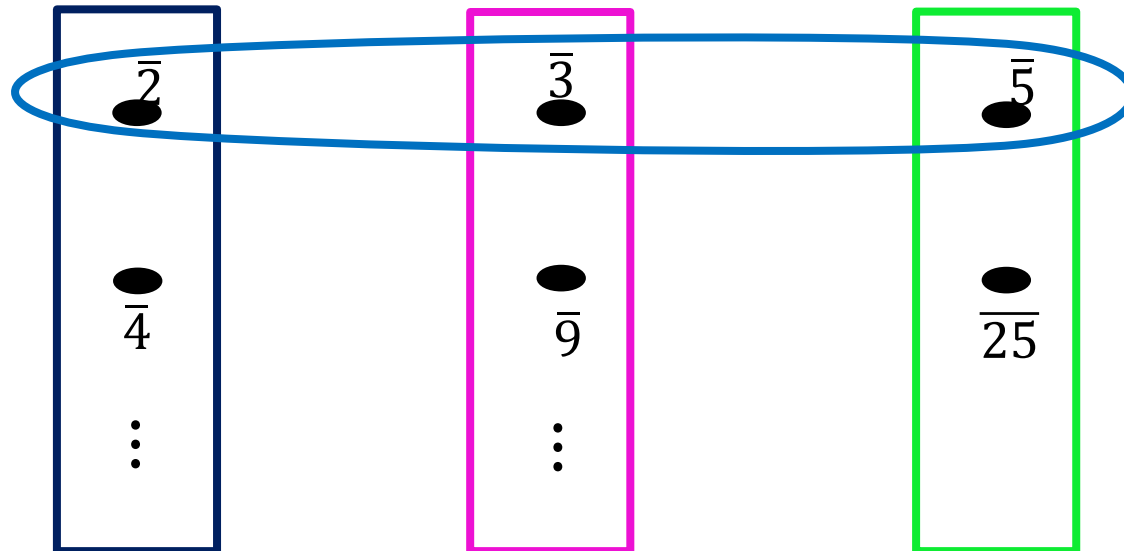
Diameter of complete k -partite k -zero-divisor hypergraph of ring R



Diameter is 2

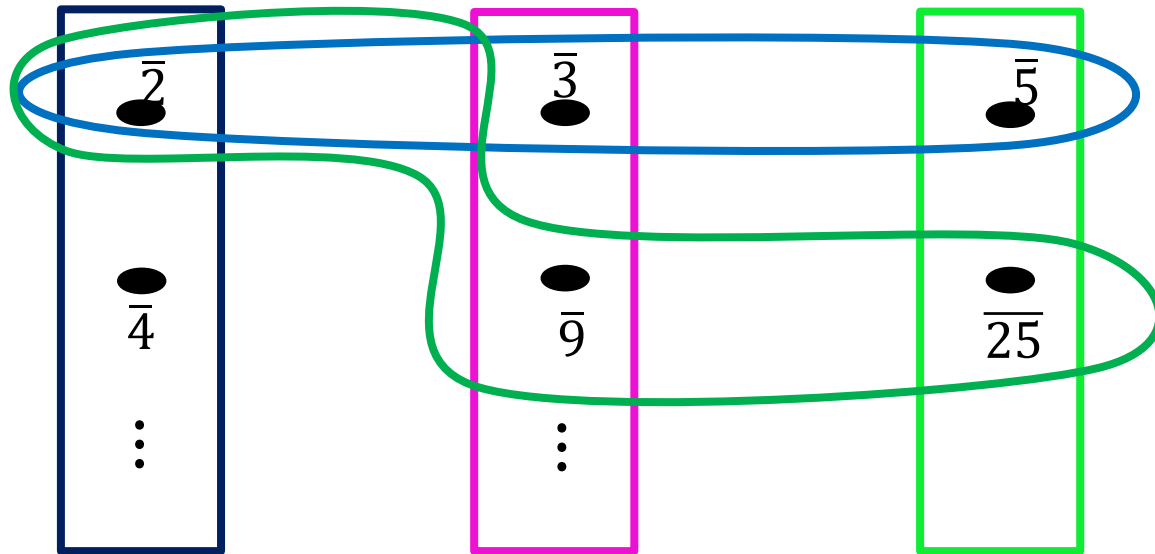
Diameter of complete k -partite k -zero-divisor hypergraph of ring R

➤ Diameter is 2



Diameter of complete k -partite k -zero-divisor hypergraph of ring R

➤ Diameter is 2



The minimum length of all cycles

The minimum length of all cycles

➤ 0 if $|Z(R/R\gamma, k)| = k$

The minimum length of all cycles

➤ 0 if $|Z(R/R\gamma, k)| = k$

Only one edge

The minimum length of all cycles

➤ 0 if $|Z(R/R\gamma, k)| = k$

Only one edge

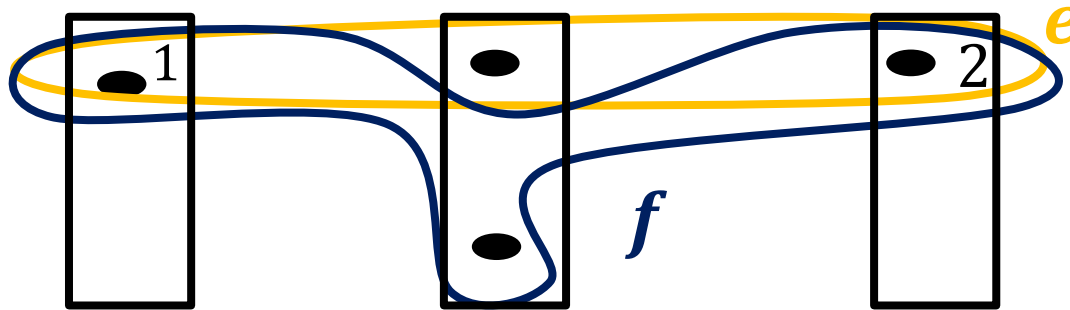
➤ 2 if $k \geq 3$ and $|Z(R/R\gamma, k)| \geq k + 1$

The minimum length of all cycles

➤ 0 if $|Z(R/R\gamma, k)| = k$

Only one edge

➤ 2 if $k \geq 3$ and $|Z(R/R\gamma, k)| \geq k + 1$



$$c = 1, e, 2, f$$

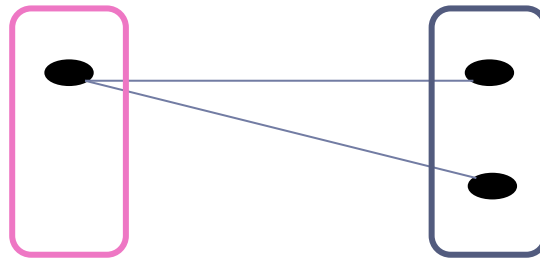
The minimum length of all cycles

The minimum length of all cycles

- 0 if $k = 2$ and $|Z(R/R\gamma, 2)| \geq 3$ (one of partite sets has only one element)

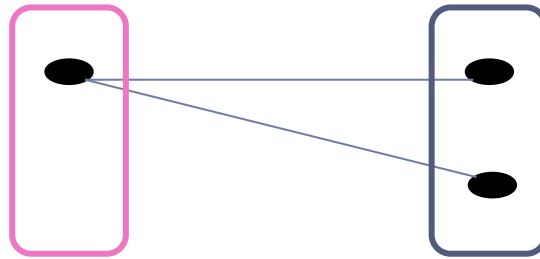
The minimum length of all cycles

- **0** if $k = 2$ and $|Z(R/R_Y, 2)| \geq 3$ (one of partite sets has only one element)



The minimum length of all cycles

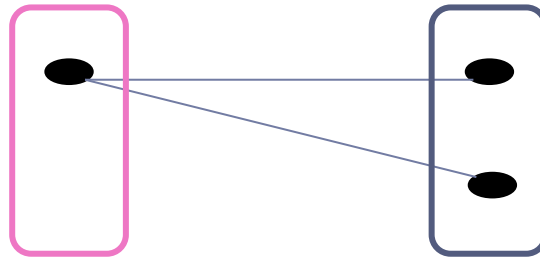
- **0** if $k = 2$ and $|Z(R/R\gamma, 2)| \geq 3$ (one of partite sets has only one element)



- **4** if $k = 2$ and $|Z(R/R\gamma, 2)| \geq 3$ (each partite set has more than one element)

The minimum length of all cycles

- **0** if $k = 2$ and $|Z(R/R\gamma, 2)| \geq 3$ (one of partite sets has only one element)



- **4** if $k = 2$ and $|Z(R/R\gamma, 2)| \geq 3$ (each partite set has more than one element)

Same idea as complete bipartite graph



Conclusion table

Conclusion table

Hypergraph	Appropriate Ideal	Vertex Set ($\gamma = p_1 p_2 p_3 \cdots p_k$)	Diameter	Minimum length of all cycles
Complete k -partite k -zero-divisor hypergraph	$Rp_1 p_2 p_3 \cdots p_k$	$V_i = Rp_i / R\gamma - \bigcup_{j \neq i} Rp_j / R\gamma$ $\bigcup_{i=1}^k V_i = Z(R / R\gamma, k)$	2	0, 2, or 4



k -partite **σ** -zero-divisor hypergraph of ring **R** where **$\sigma \geq k$**



k -partite **σ** -zero-divisor hypergraph of ring **R** where **$\sigma \geq k$**

How to construct **k** -partite **σ** -zero-divisor hypergraph

k -partite σ -zero-divisor hypergraph of ring R where $\sigma \geq k$

How to construct k -partite σ -zero-divisor hypergraph

Construct complete k -
partite k -zero-divisor
hypergraph

k -partite σ -zero-divisor hypergraph of ring R where $\sigma \geq k$

How to construct k -partite σ -zero-divisor hypergraph

Construct complete k -
partite k -zero-divisor
hypergraph

Construct complete l -zero-
divisor hypergraph

3-partite 4-zero-divisor hypergraph of ring R



3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

$$Rp_1p_2p_3$$

3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

$Rp_1p_2p_3$



complete 2-zero-divisor
hypergraph

3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

$Rp_1p_2p_3$



complete 2-zero-divisor
hypergraph

Rp^2

3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

complete 2-zero-divisor
hypergraph

$$Rp_1p_2p_3 + Rp^2$$

$$Rp_1^2p_2p_3$$

3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

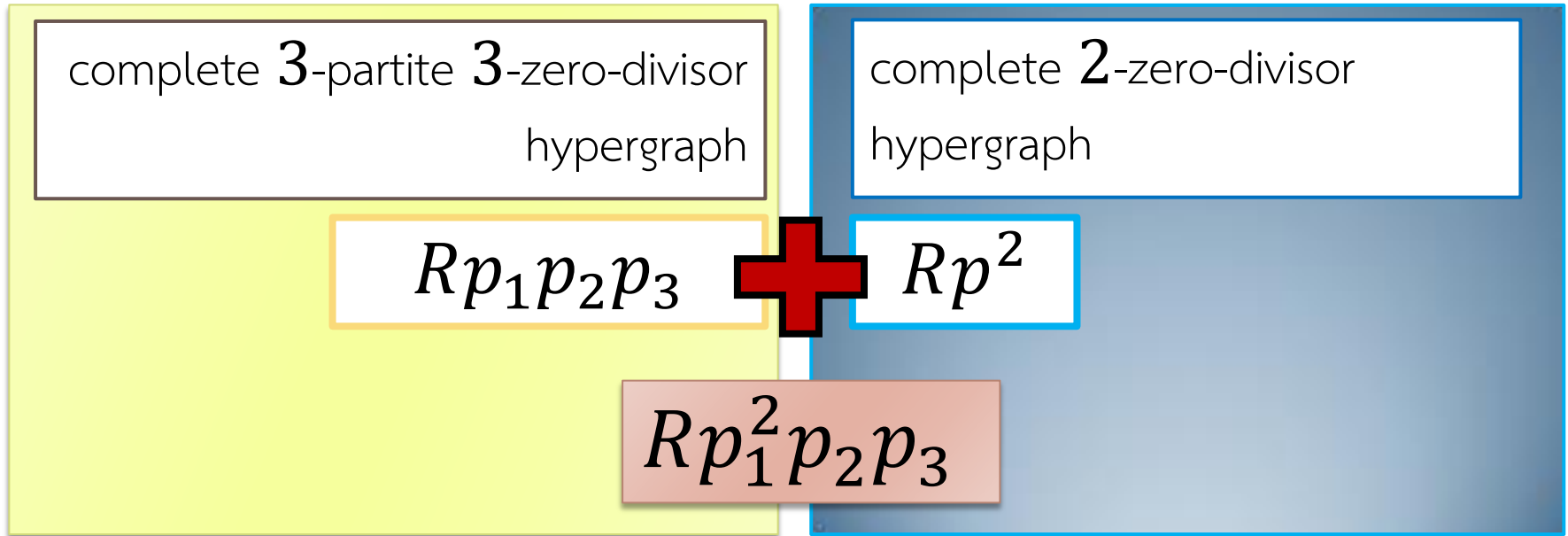
complete 2-zero-divisor
hypergraph

$$Rp_1p_2p_3 + Rp^2$$

$$Rp_1^2p_2p_3$$

$$\text{Ring } \mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$$

3-partite 4-zero-divisor hypergraph of ring R



Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

$$\begin{array}{c} \bar{2} \\ \bullet \\ \hline \bar{14} \end{array}$$

$$\begin{array}{c} \bar{3} \\ \bullet \\ \hline \bar{9} \end{array}$$

$$\begin{array}{c} \bar{5} \\ \bullet \\ \hline \bar{35} \end{array}$$

3-partite 4-zero-divisor hypergraph of ring R

complete 3-partite 3-zero-divisor
hypergraph

$$Rp_1p_2p_3$$

Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

$\bar{2}$

$\frac{\bullet}{14}$

$\bar{3}$

$\frac{\bullet}{9}$

$\bar{5}$

$\frac{\bullet}{35}$



3-partite 4-zero-divisor hypergraph of ring R

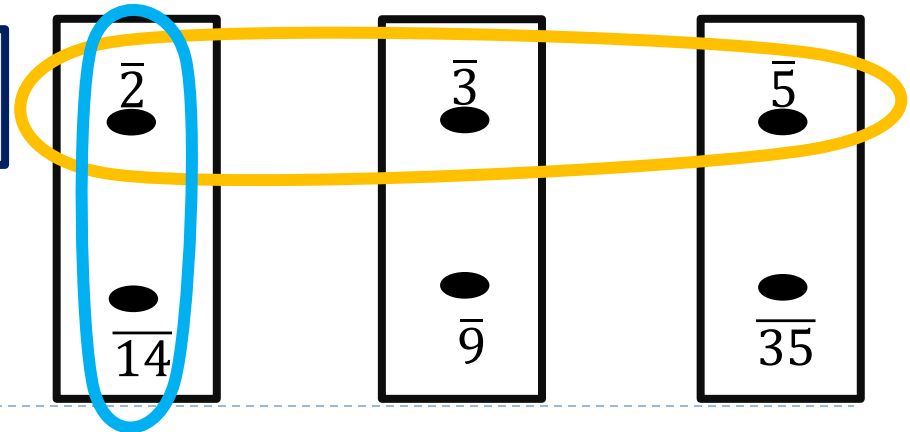
complete 3-partite 3-zero-divisor
hypergraph

$$Rp_1p_2p_3$$

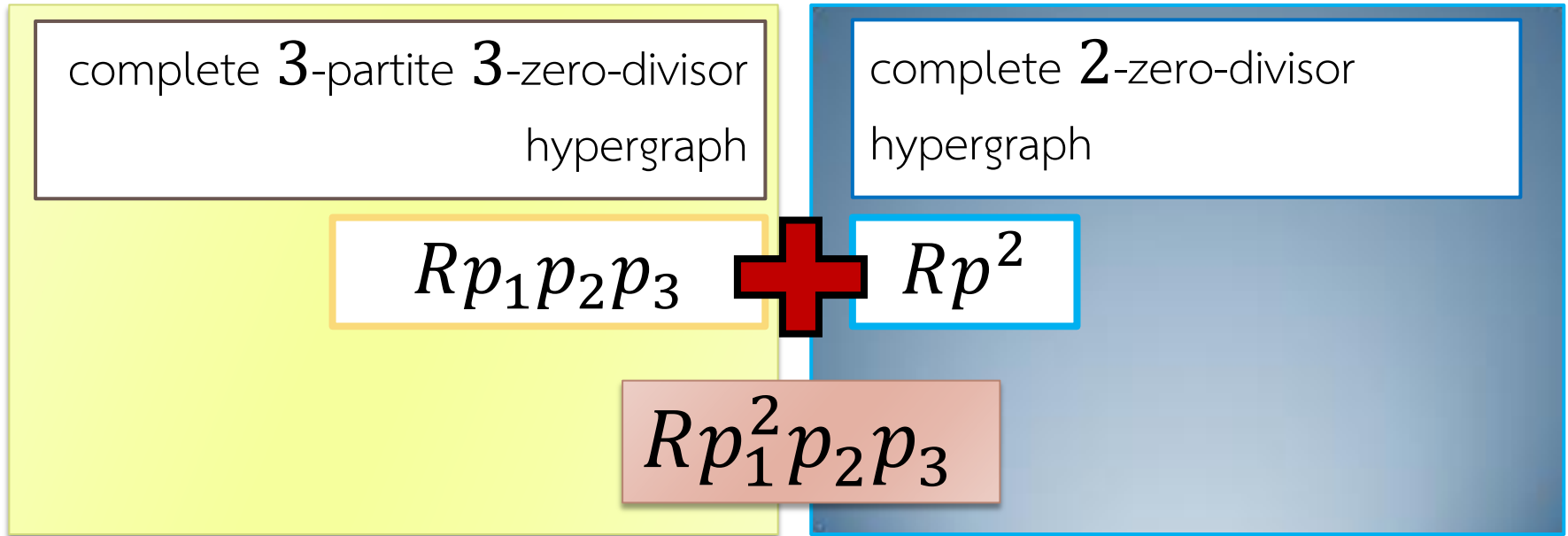
complete 2-zero-divisor
hypergraph

$$Rp^2$$

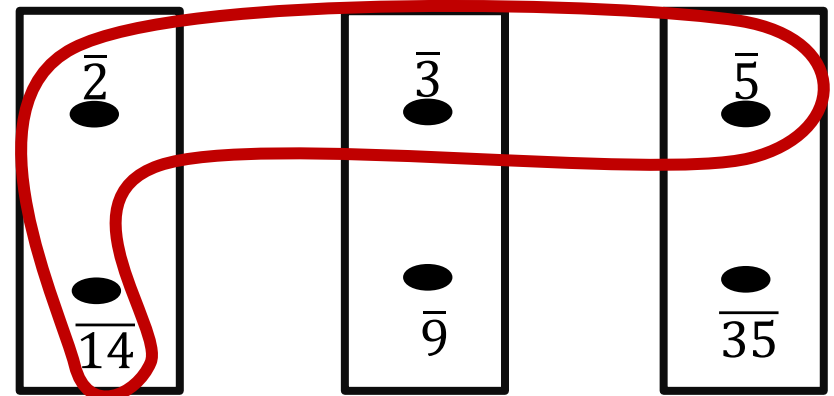
Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$



3-partite 4-zero-divisor hypergraph of ring R



Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$



k -partite **σ** -zero-divisor hypergraph of ring **R** where **$\sigma \geq k$**



k -partite σ -zero-divisor hypergraph of ring R where $\sigma \geq k$

Commutative ring: $R / Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Appropriate ideal: $Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Condition: R has at least k prime elements



k -partite σ -zero-divisor hypergraph of ring R where $\sigma \geq k$

$$\sigma = \sum_{m=1}^k \alpha_m$$

Commutative ring: $R / Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Condition: R has at least k prime elements

Appropriate ideal: $Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

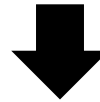
k -partite **σ** -zero-divisor hypergraph of ring R where $\sigma \geq k$

$$\sigma = \sum_{m=1}^k \alpha_m$$

Commutative ring: $R / Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Condition: R has at least k prime elements

Appropriate ideal: $Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$



Let $\pi = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

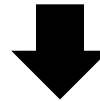
k -partite σ -zero-divisor hypergraph of ring R where $\sigma \geq k$

$$\sigma = \sum_{m=1}^k \alpha_m$$

Commutative ring: $R / Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Condition: R has at least k prime elements

Appropriate ideal: $Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$



Let $\pi = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Each partite set V_i :

- $Rp_i / R\pi - \bigcup_{j \neq i} Rp_j / R\pi$ if $\alpha_i = 1$
- $Rp_i / R\pi - \left(Rp_i^2 / R\pi \cup \bigcup_{j \neq i} Rp_j / R\pi \right)$ if $\alpha_i \geq 2$

$$\bigcup_{i=1}^k V_i = Z(R / R\pi, \sigma)$$

k -partite σ -zero-divisor hypergraph of ring R where $\sigma \geq k$

$$\sigma = \sum_{m=1}^k \alpha_m$$

Commutative ring: $R / Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Condition: R has at least k prime elements

Appropriate ideal: $Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Let $\pi = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$

Each partite set V_i :

- $Rp_i / R\pi - \bigcup_{j \neq i} Rp_j / R\pi$ if $\alpha_i = 1$
- $Rp_i / R\pi - \left(Rp_i^2 / R\pi \cup \bigcup_{j \neq i} Rp_j / R\pi \right)$ if $\alpha_i \geq 2$

$$\bigcup_{i=1}^k V_i = Z(R / R\pi, \sigma)$$

k -partite σ -zero-divisor hypergraph



Example of **3**-partite **4**-zero-divisor hypergraph of ring R



Example of **3**-partite **4**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

Example of **3**-partite **4**-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$$Z(\mathbb{Z}_{60}, 4) = \{\bar{2}, \bar{3}, \bar{5}, \bar{9}, \bar{14}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{33}, \bar{34}, \bar{35}, \bar{38}, \bar{39}, \bar{46}, \bar{51}, \bar{55}, \bar{57}, \bar{58}\}$$

Example of **3**-partite **4**-zero-divisor hypergraph of ring R

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$\bar{2}$

$\bar{3}$

$\bar{5}$

$\bar{14}$

$\bar{9}$

$\bar{35}$

\vdots

\vdots

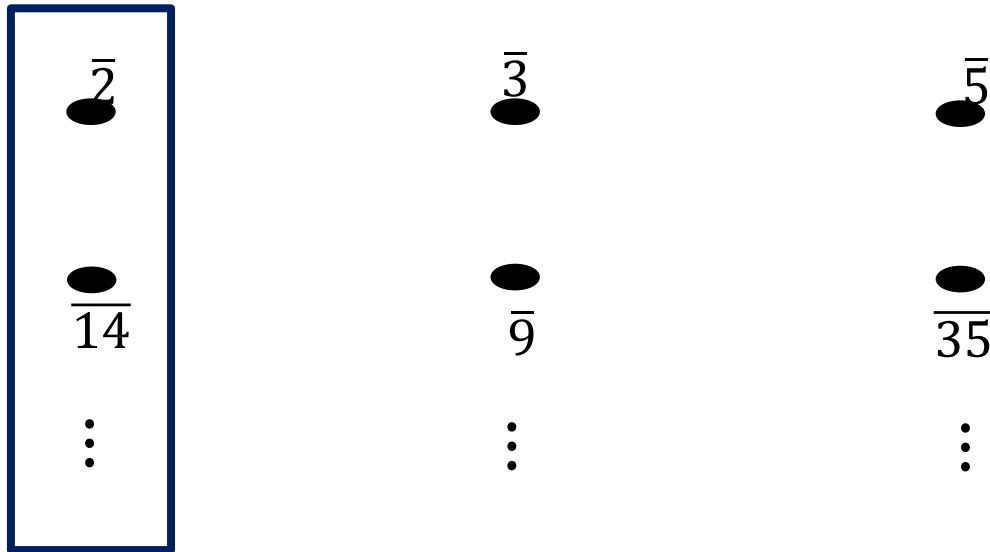
\vdots

Example of 3-partite 4-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$$Z(\mathbb{Z}_{60}, 4) = \{\bar{2}, \bar{3}, \bar{5}, \bar{9}, \bar{14}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{33}, \bar{34}, \bar{35}, \bar{38}, \bar{39}, \bar{46}, \bar{51}, \bar{55}, \bar{57}, \bar{58}\}$$



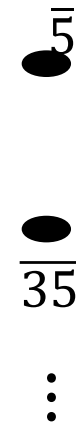
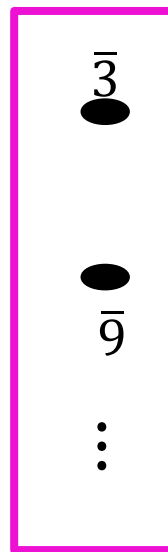
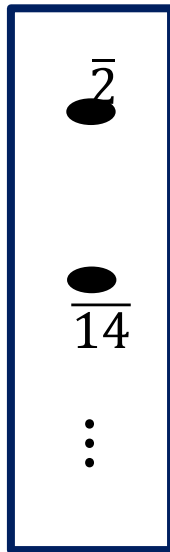
$$V_1 = \{\bar{2}, \bar{14}, \bar{22}, \bar{26}, \bar{34}, \bar{38}, \bar{46}, \bar{58}\}$$

Example of 3-partite 4-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$$Z(\mathbb{Z}_{60}, 4) = \{\bar{2}, \bar{3}, \bar{5}, \bar{9}, \bar{14}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{33}, \bar{34}, \bar{35}, \bar{38}, \bar{39}, \bar{46}, \bar{51}, \bar{55}, \bar{57}, \bar{58}\}$$



$$V_1 = \{\bar{2}, \bar{14}, \bar{22}, \bar{26}, \bar{34}, \bar{38}, \bar{46}, \bar{58}\}$$

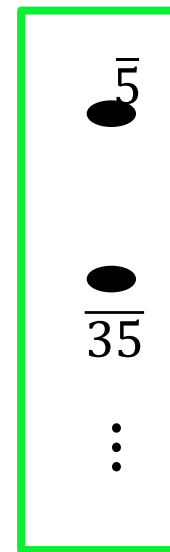
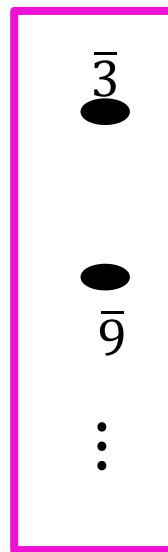
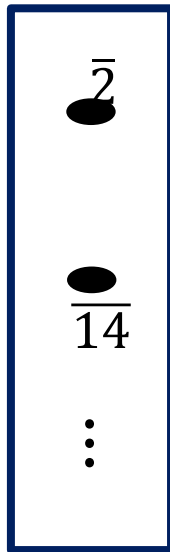
$$V_2 = \{\bar{3}, \bar{9}, \bar{21}, \bar{27}, \bar{33}, \bar{39}, \bar{51}, \bar{57}\}$$

Example of 3-partite 4-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

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$$V_3 = \{\bar{5}, \bar{25}, \bar{35}, \bar{55}\}$$

$$V_1 = \{\bar{2}, \bar{14}, \bar{22}, \bar{26}, \bar{34}, \bar{38}, \bar{46}, \bar{58}\}$$

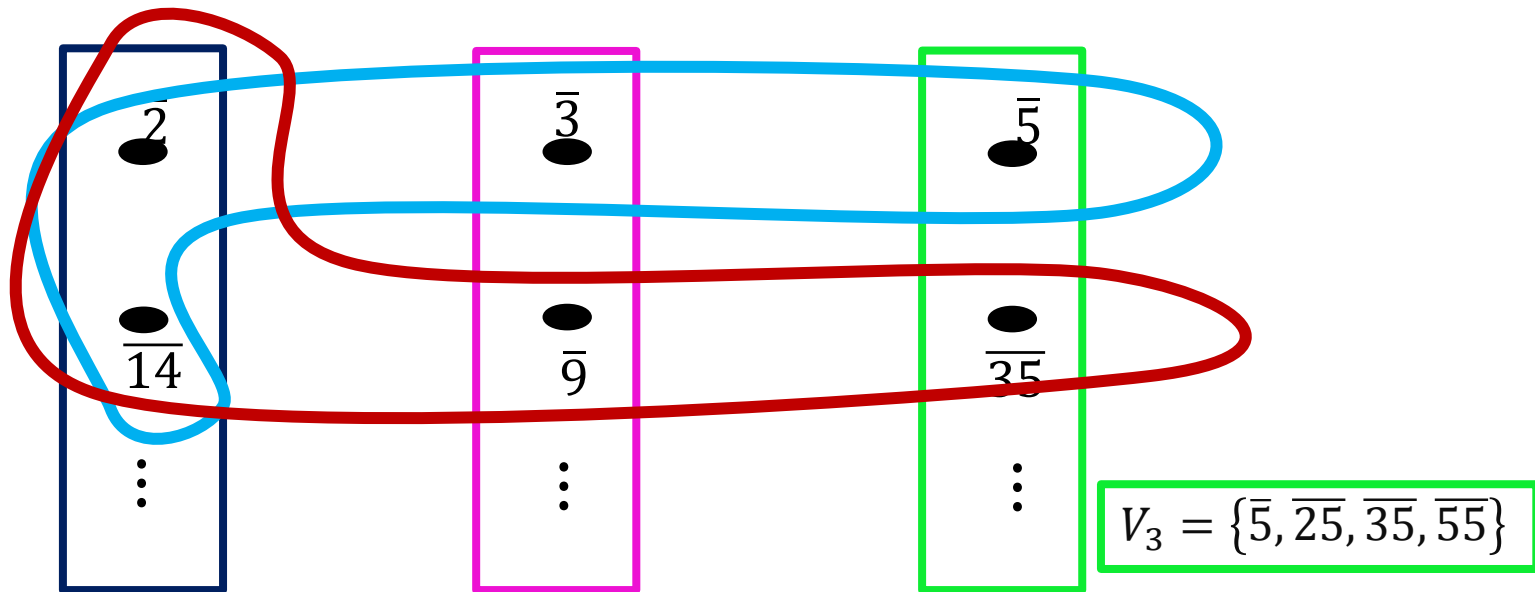
$$V_2 = \{\bar{3}, \bar{9}, \bar{21}, \bar{27}, \bar{33}, \bar{39}, \bar{51}, \bar{57}\}$$

Example of 3-partite 4-zero-divisor hypergraph of ring R

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$$Z(\mathbb{Z}_{60}, 4) = \{\bar{2}, \bar{3}, \bar{5}, \bar{9}, \bar{14}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{33}, \bar{34}, \bar{35}, \bar{38}, \bar{39}, \bar{46}, \bar{51}, \bar{55}, \bar{57}, \bar{58}\}$$



$$V_1 = \{\bar{2}, \bar{14}, \bar{22}, \bar{26}, \bar{34}, \bar{38}, \bar{46}, \bar{58}\}$$

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Diameter of k -partite σ -zero-divisor hypergraph of ring R

Diameter of k -partite σ -zero-divisor hypergraph of ring R



Diameter is 2

Diameter of k -partite σ -zero-divisor hypergraph of ring R

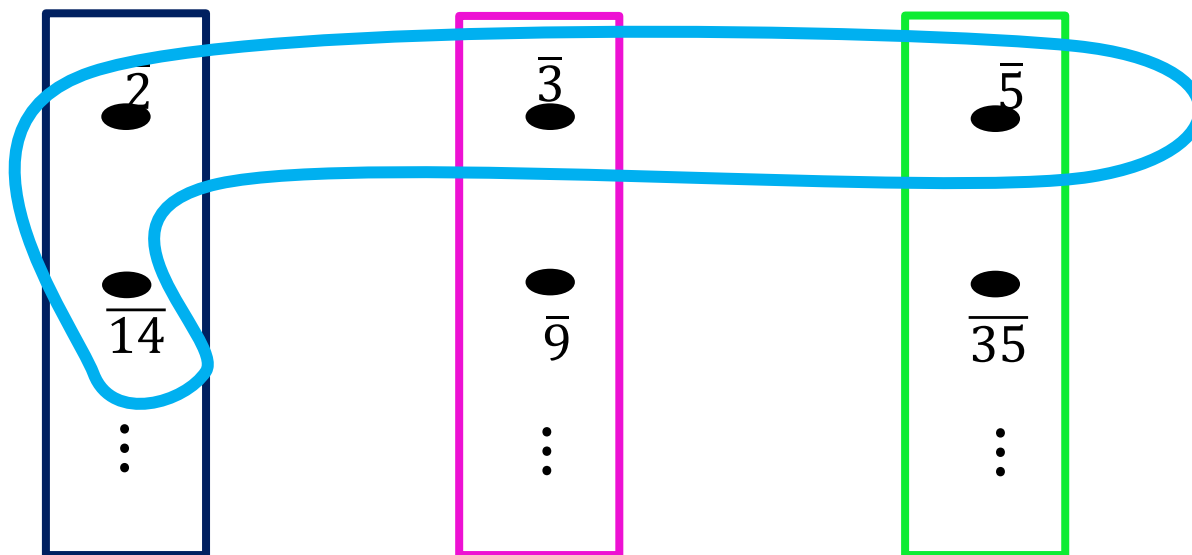
➤ Diameter is 2

$$\mathbb{Z}_{60} \cong \mathbb{Z} / (2^2 \cdot 3 \cdot 5)\mathbb{Z}$$

Diameter of k -partite σ -zero-divisor hypergraph of ring R

➤ Diameter is 2

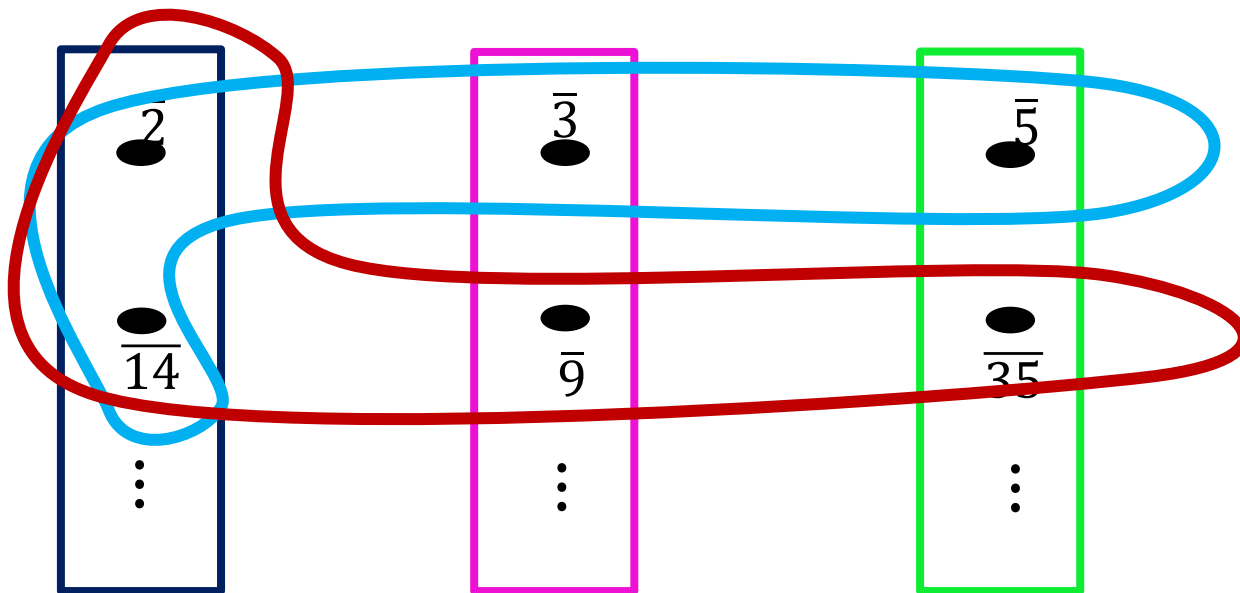
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Diameter of k -partite σ -zero-divisor hypergraph of ring R

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The minimum length of all cycles

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➤ 0 if $|Z(R/R\pi, \sigma)| = \sigma$

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Only one edge

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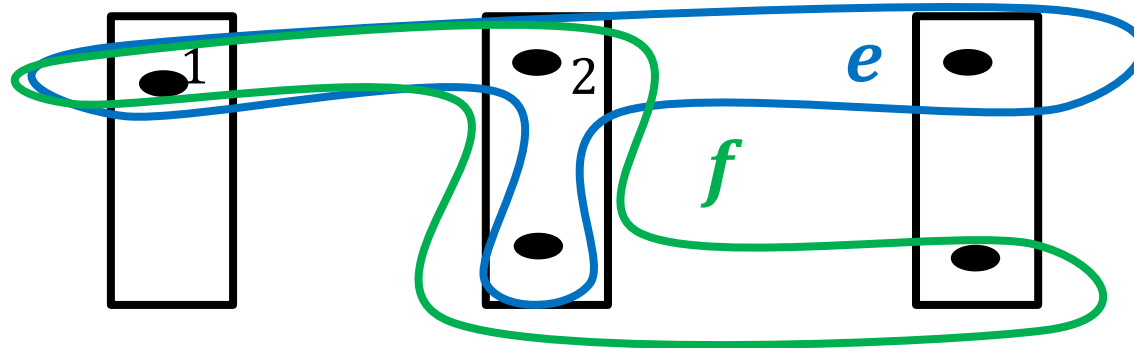
➤ 2 if $k \geq 3$ and $|Z(R/R\pi, \sigma)| \geq \sigma + 1$

The minimum length of all cycles

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Only one edge

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$c = 1, e, 2, f$

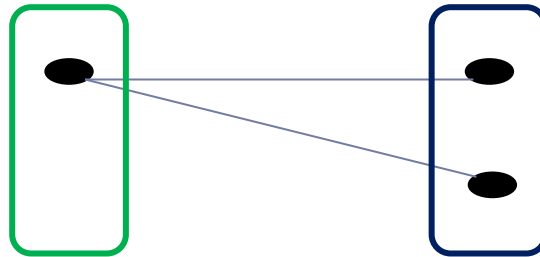
The minimum length of all cycles

The minimum length of all cycles

➤ **0** if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$
and $|V_1| = 1$ and $\alpha_2 = 1$

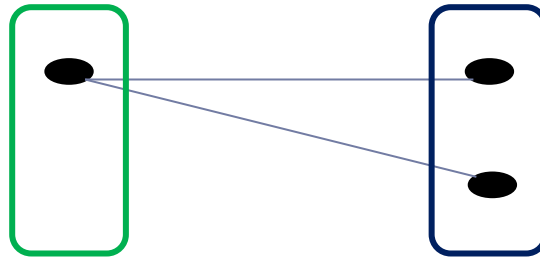
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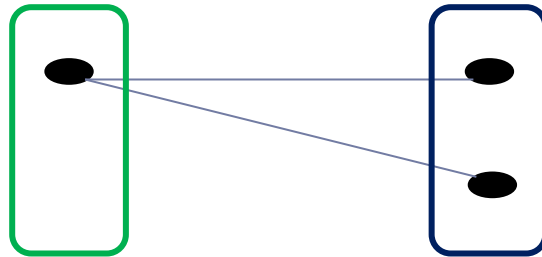
➤ 0 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$
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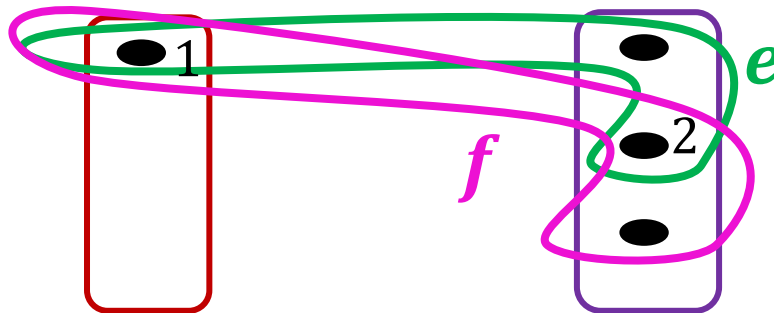
➤ 2 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$
and $|V_1| = 1$ and $\alpha_2 \geq 2$

The minimum length of all cycles

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$c = 1, e, 2, f$

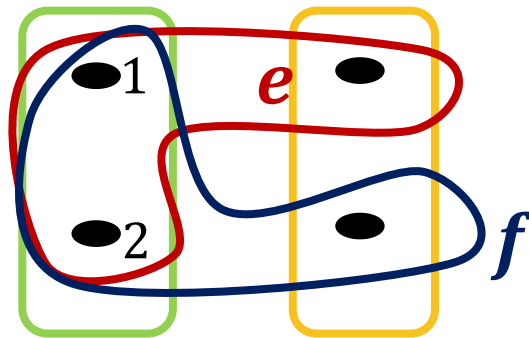
The minimum length of all cycles

The minimum length of all cycles

- 2 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$

The minimum length of all cycles

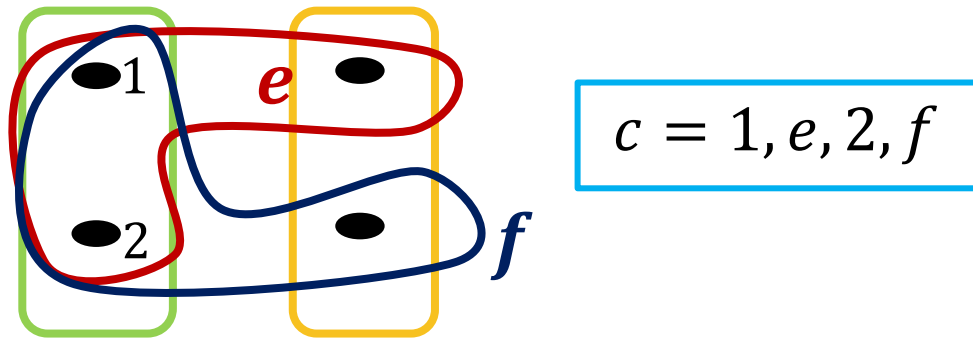
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$$c = 1, e, 2, f$$

The minimum length of all cycles

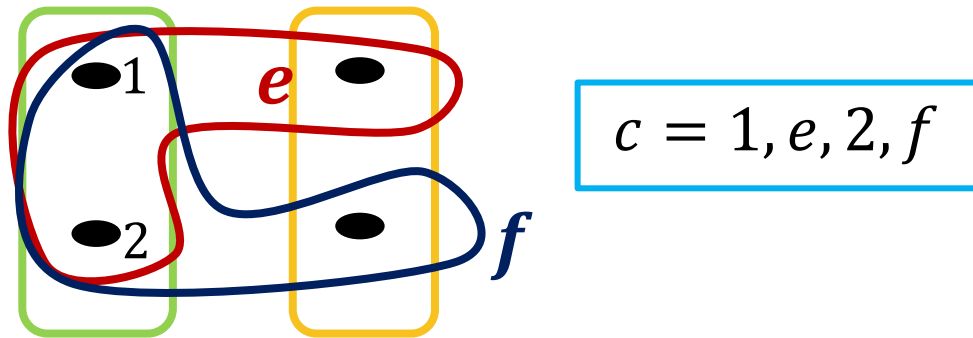
- 2 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$



- 4 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ with $\alpha_i = 1$ for all $1 \leq i \leq 2$

The minimum length of all cycles

- 2 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$



- 4 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ with $\alpha_i = 1$ for all $1 \leq i \leq 2$

Same idea as complete bipartite graph

Conclusion table

Conclusion table

Hypergraph	Appropriate Ideal	Vertex Set $(\sigma = \sum_{m=1}^k \alpha_m,$ $\pi = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k})$	Diameter	Minimum length of all cycles
k -partite σ -zero-divisor hypergraph	$Rp_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \dots p_k^{\alpha_k}$	$V_i = Rp_i/R\pi - \bigcup_{j \neq i} Rp_j/R\pi$ if $\alpha_i = 1$ $V_i = Rp_i/R\pi - (Rp_i^2/R\pi \cup \bigcup_{j \neq i} Rp_j/R\pi)$ if $\alpha_i \geq 2$ $\bigcup_{i=1}^k V_i = Z(R/R\pi, \sigma)$	2	0, 2, or 4





Recent Works



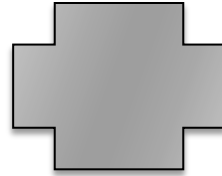


Games played on hypergraphs



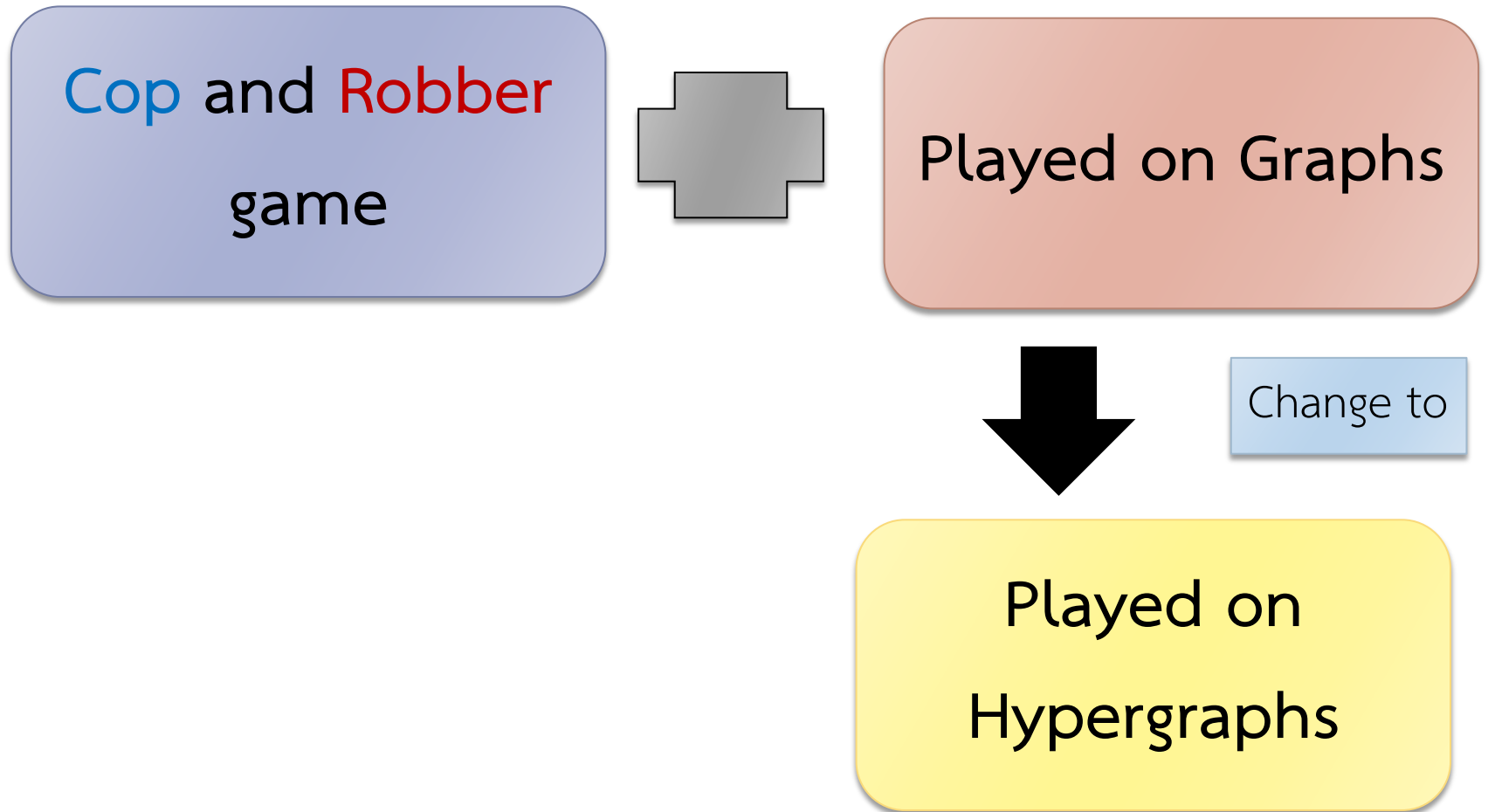
Games played on hypergraphs

Cop and Robber
game



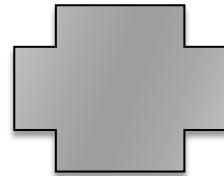
Played on Graphs

Games played on hypergraphs



Games played on hypergraphs

Cop and Robber
game



Played on
Hypergraphs



Cop and Robber game played on graphs

Nowakowski and Winkler



Cop and Robber game played on graphs



Two players

Cop and Robber game played on graphs

Two players



Cop

Cop and Robber game played on graphs

Two players



Cop



Robber

Cop and Robber game played on graphs

Two players



Cop



Robber

Rules of the game played on graphs

Cop and Robber game played on graphs

Two players



Cop



Robber

Rules of the game played on graphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex

Cop and Robber game played on graphs

Two players



Cop



Robber

Rules of the game played on graphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex

2. In each round, the cop and the robber take alternatively moving from their present vertex to other vertices along edges or staying put.

Nowakowski and Winkler

Cop and Robber game played on graphs



Cop and Robber game played on graphs

How to finish the game

Cop and Robber game played on graphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves

Cop and Robber game played on graphs

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Cop and Robber game played on graphs

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Robber wins if robber can run away

Cop and Robber game played on graphs

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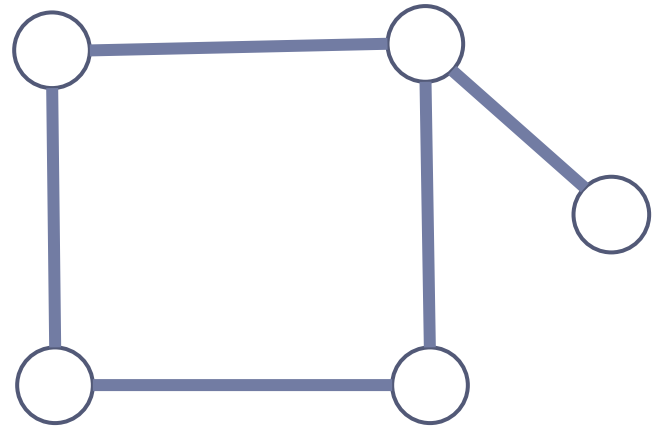
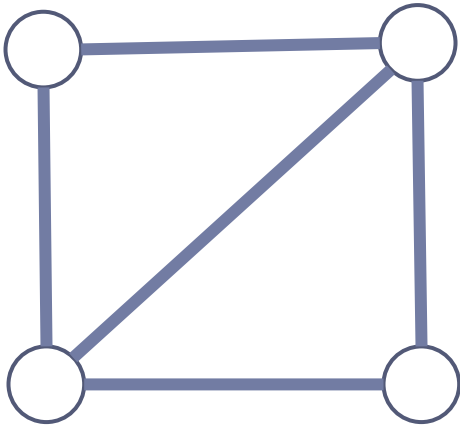


Example of cop wins and robber wins



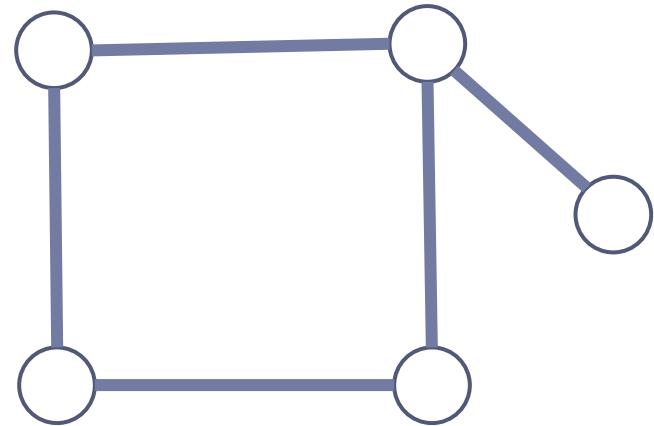
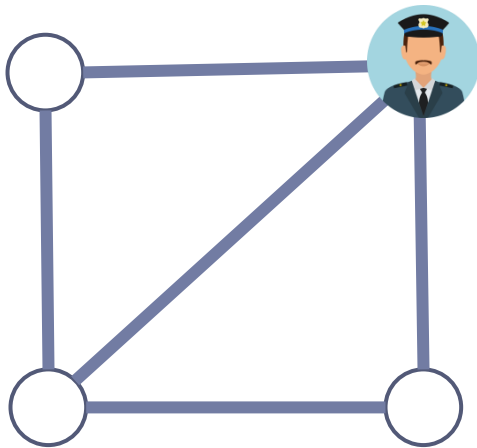
Example of cop wins and robber wins

Given two graphs, which one that cop wins?



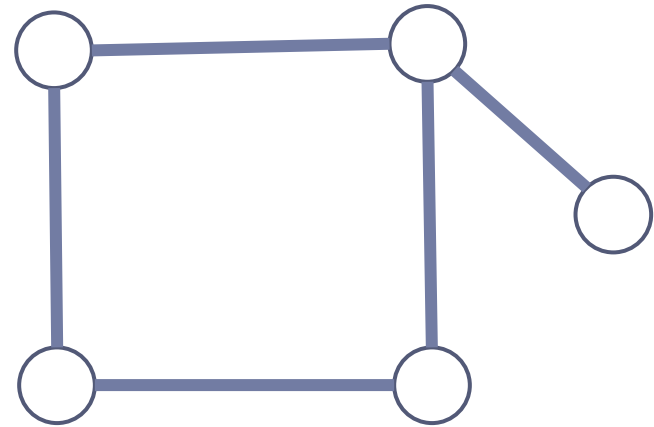
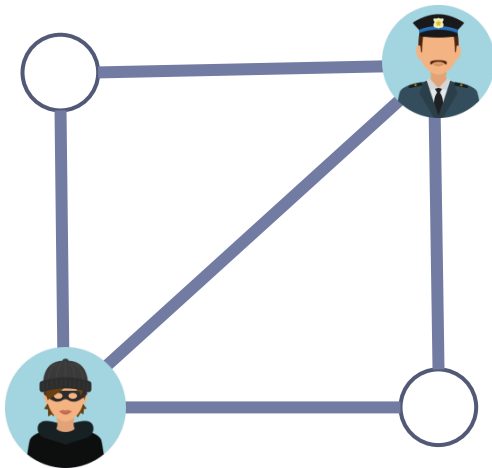
Example of **cop** wins and **robber** wins

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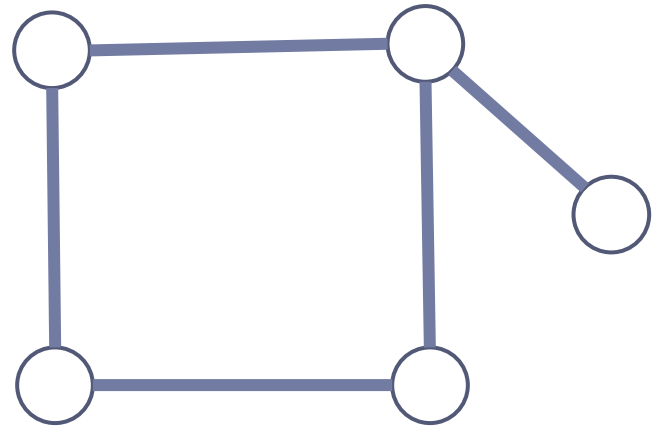
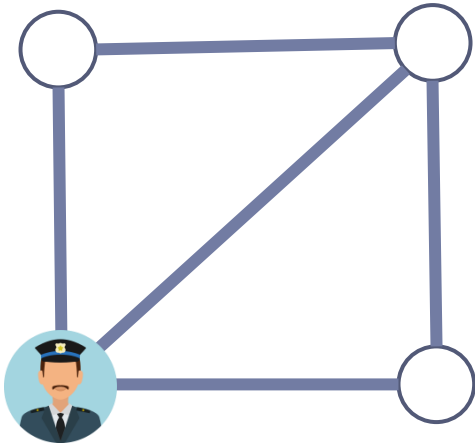
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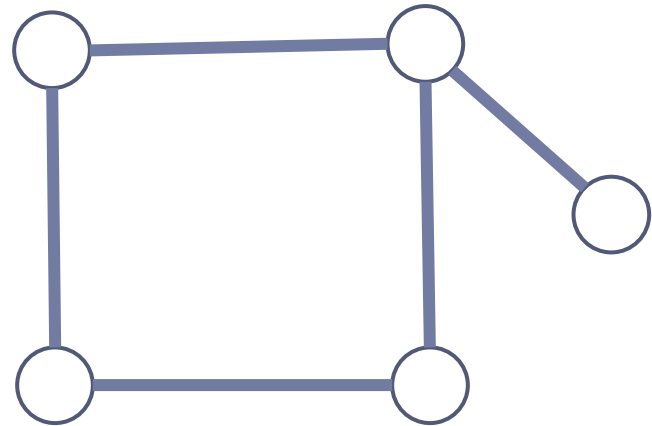
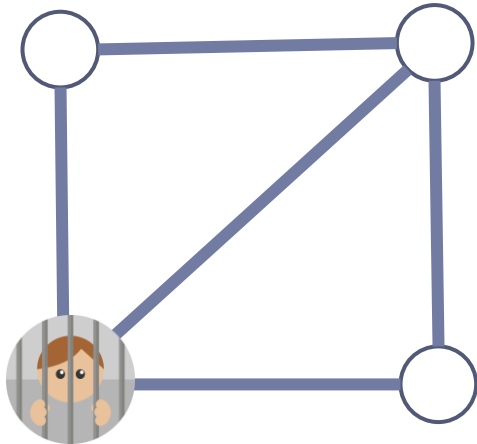
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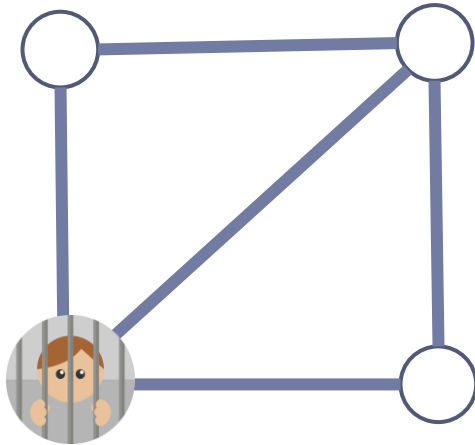
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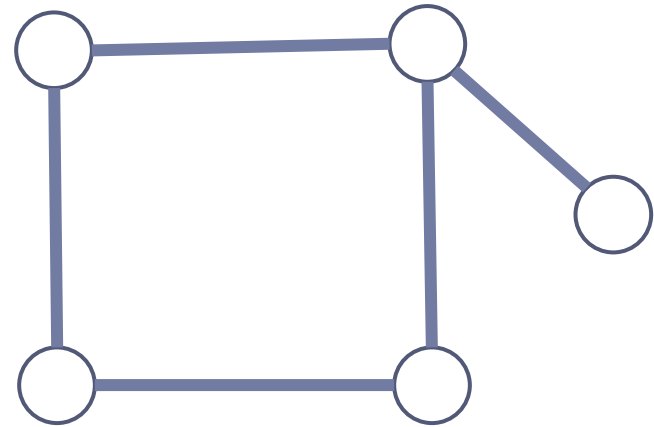


Example of cop wins and robber wins

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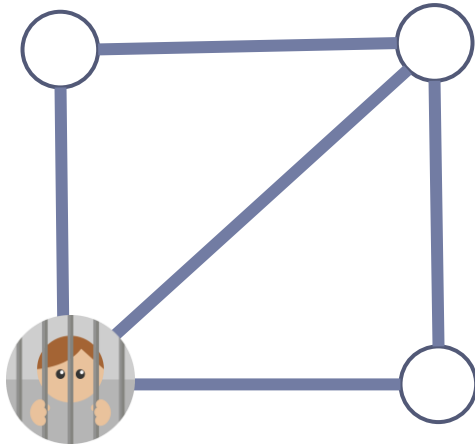


COP WINS

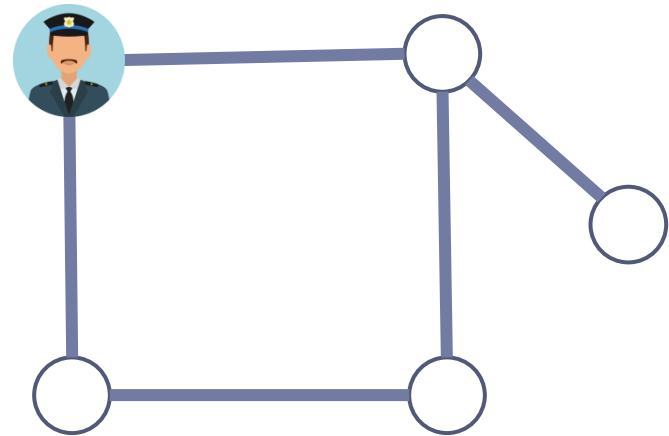


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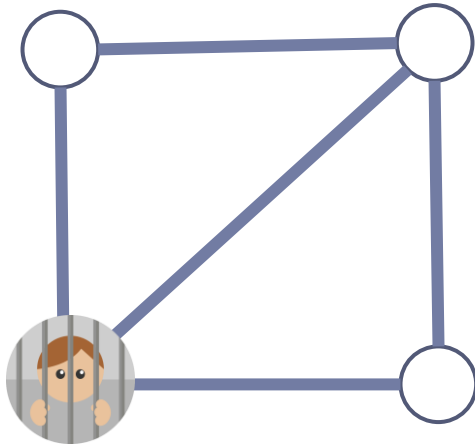


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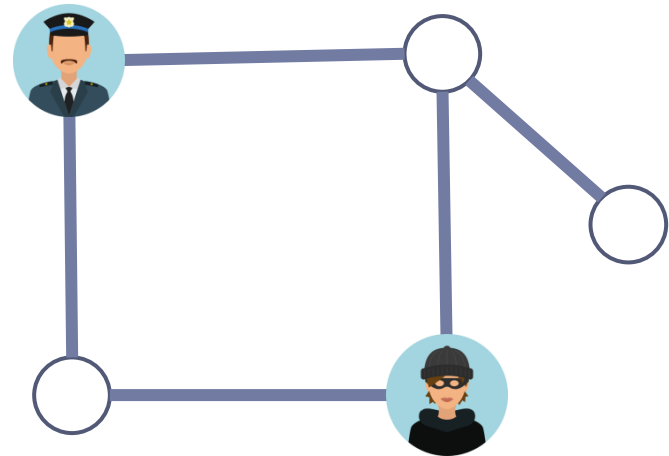


Example of **cop** wins and **robber** wins

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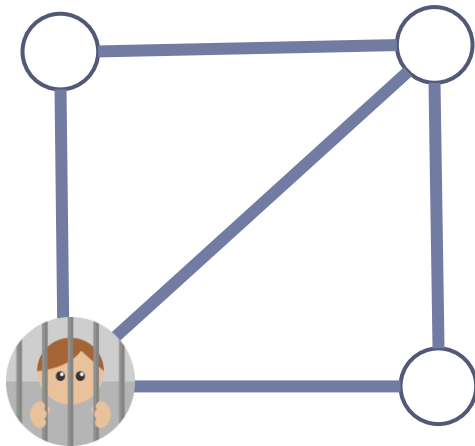


COP WINS

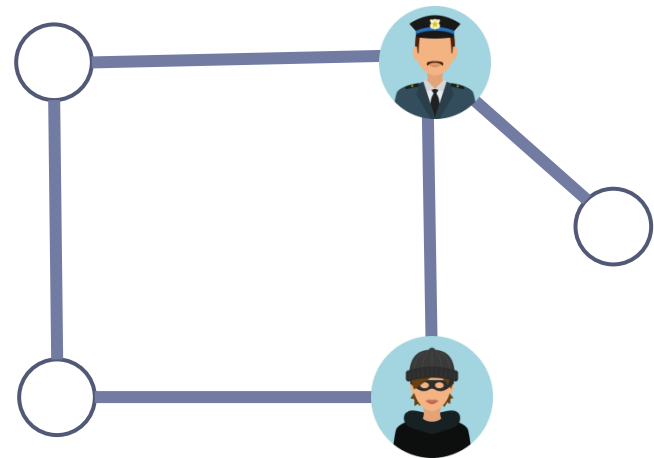


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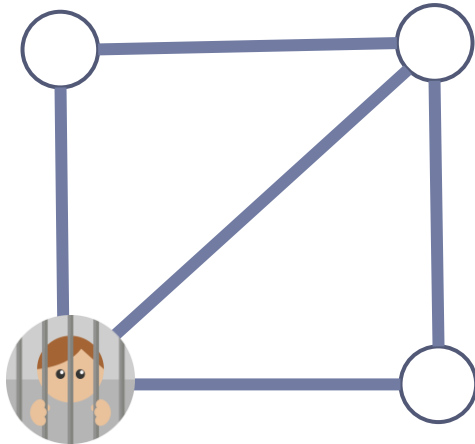


COP WINS

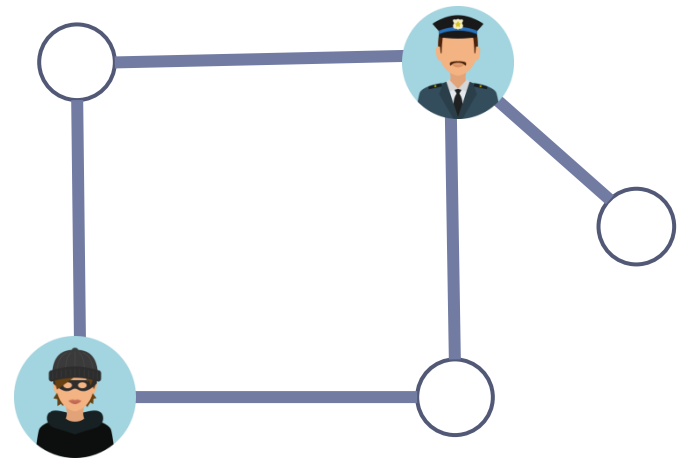


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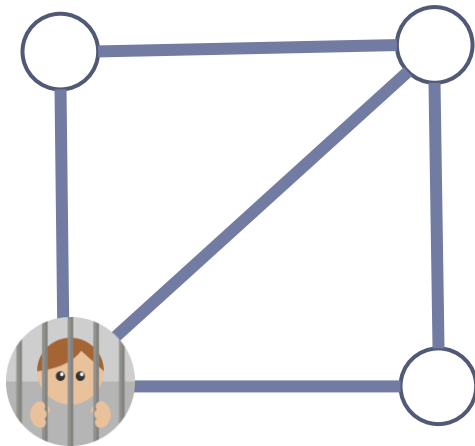


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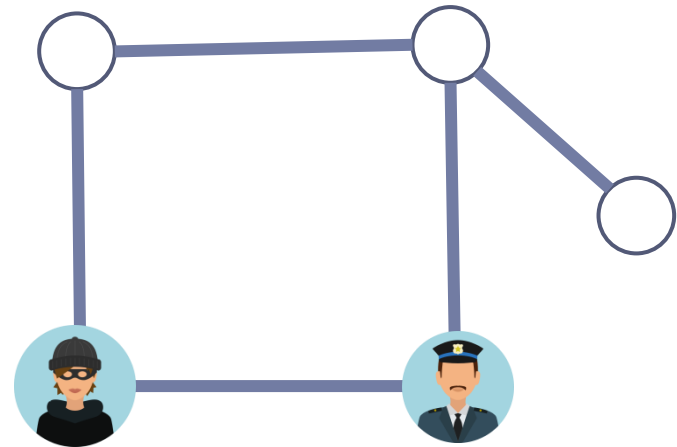


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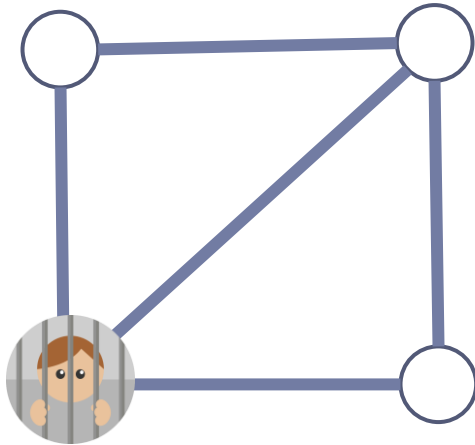


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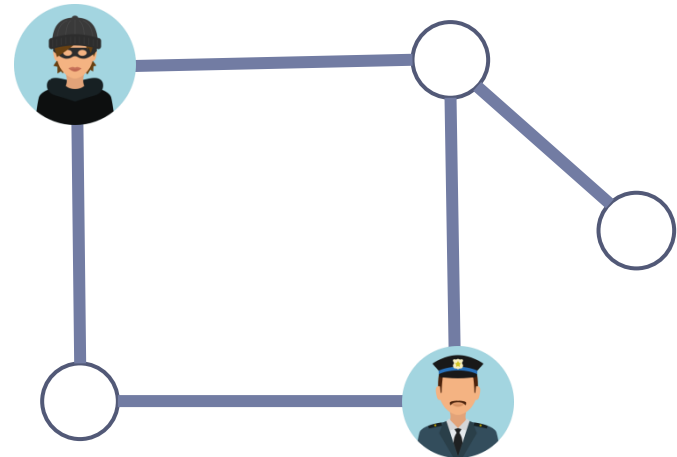


Example of **cop** wins and **robber** wins

Given two graphs, which one that **cop** wins?

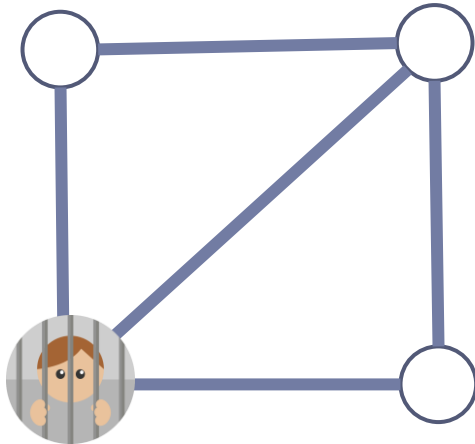


COP WINS

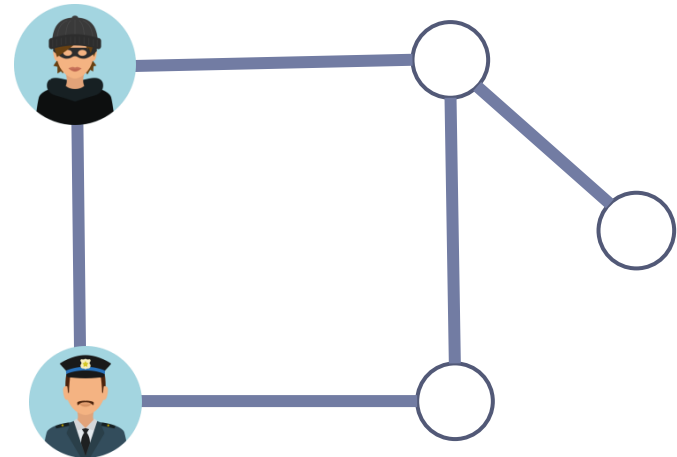


Example of cop wins and robber wins

Given two graphs, which one that cop wins?

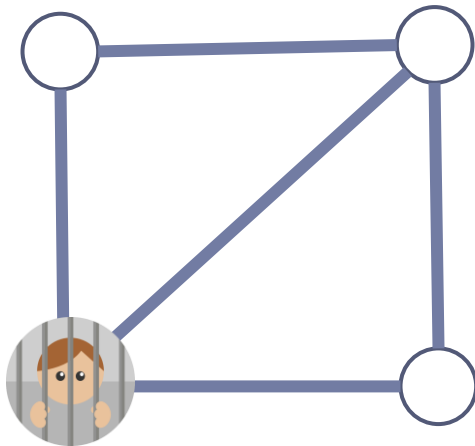


COP WINS

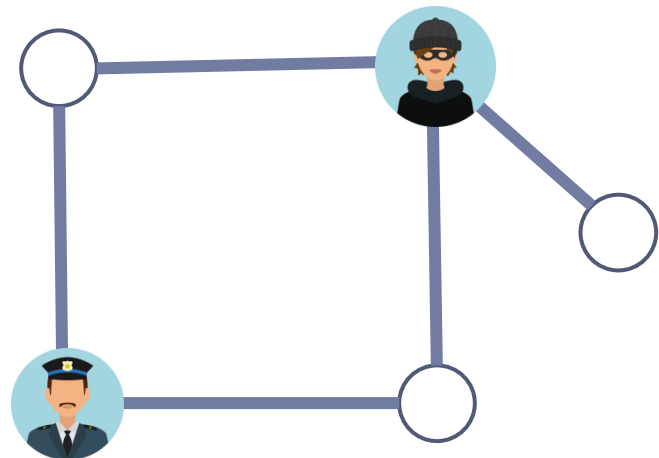


Example of **cop** wins and **robber** wins

Given two graphs, which one that **cop** wins?

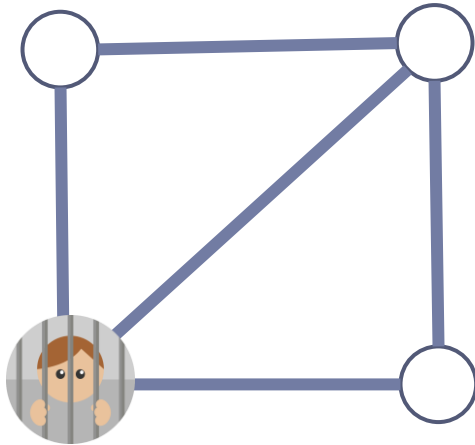


COP WINS

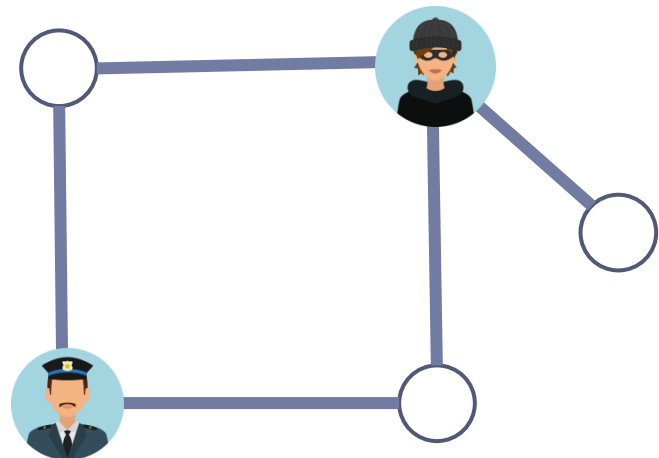


Example of **cop** wins and **robber** wins

Given two graphs, which one that **cop** wins?



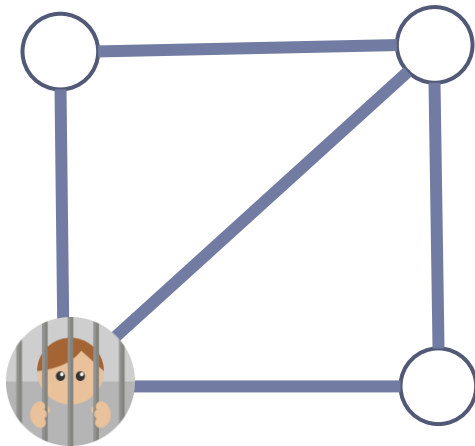
COP WINS



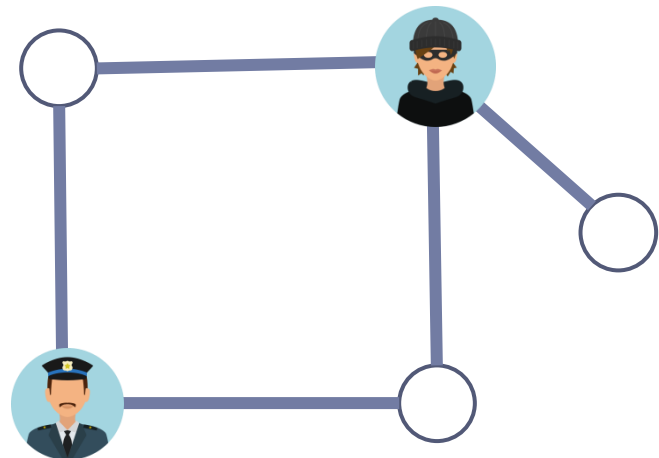
ROBBER WINS

Example of **cop** wins and **robber** wins

Given two graphs, which one that **cop** wins?



COP WINS



ROBBER WINS

The graph which cop wins is called a **cop-win graph**; otherwise, a **robber-win graph**.



Cop and Robber game played on hypergraphs

Cop and Robber game played on hypergraphs

Two players



Cop



Robber

Cop and Robber game played on hypergraphs

Two players



Cop



Robber

Rules of the game played on hypergraphs

Cop and Robber game played on hypergraphs

Two players



Rules of the game played on hypergraphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex

Cop and Robber game played on hypergraphs

Two players



Cop



Robber

Rules of the game played on hypergraphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex
2. In each round, they take alternatively moving from their present vertex x to any vertex y belonging to the same hyperedge as vertex x or staying put.

Cop and Robber game played on hypergraphs

Cop and Robber game played on hypergraphs

How to finish the game

Cop and Robber game played on hypergraphs

How to finish the game

Same as playing on graphs

Cop and Robber game played on hypergraphs

How to finish the game

Same as playing on graphs

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves. The hypergraph which cop wins is called a **cop-win hypergraph**

Cop and Robber game played on hypergraphs

How to finish the game

Same as playing on graphs

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves. The hypergraph which cop wins is called a **cop-win hypergraph**

Robber wins if robber can run away. The hypergraph which robber wins is called a **robber-win hypergraph**

Cop and Robber game played on hypergraphs



Cop and Robber game played on hypergraphs

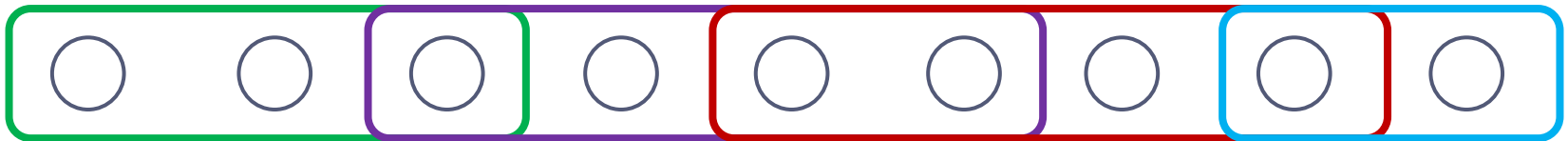
Baird

A path is a cop-win hypergraph

Cop and Robber game played on hypergraphs

Baird

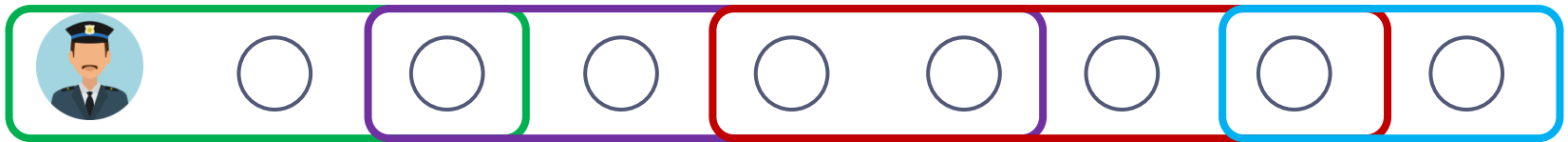
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

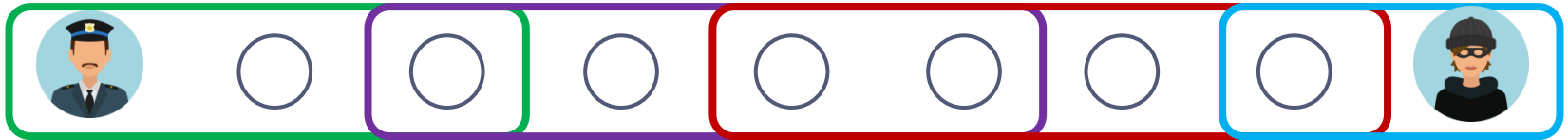
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

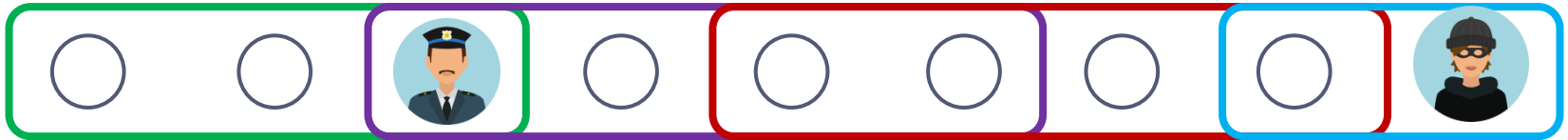
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

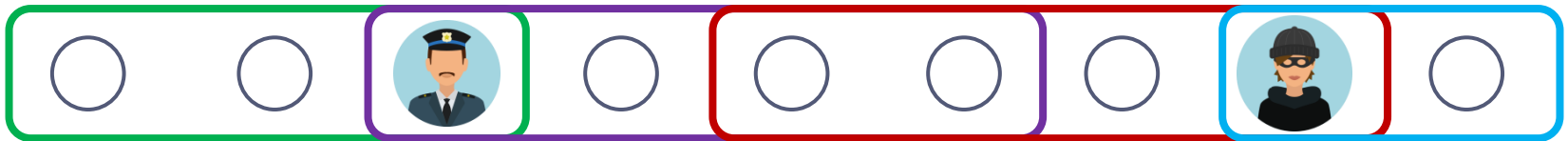
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

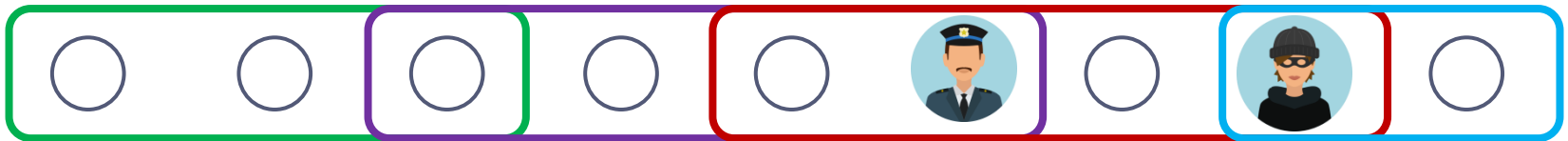
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

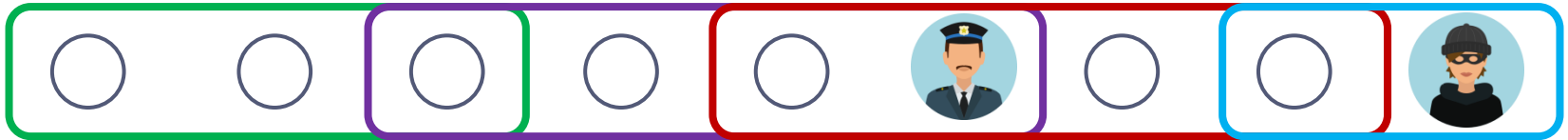
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

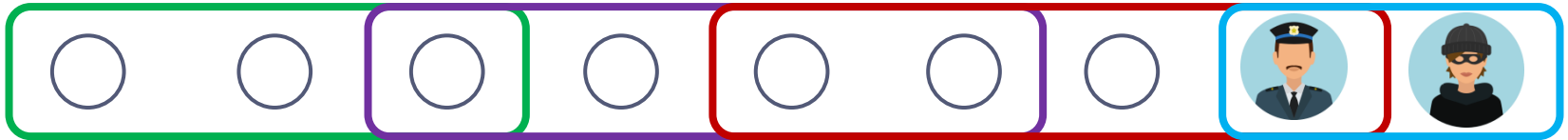
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

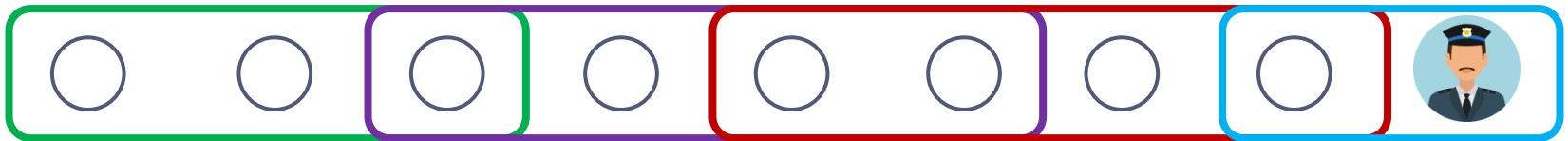
A path is a cop-win hypergraph



Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

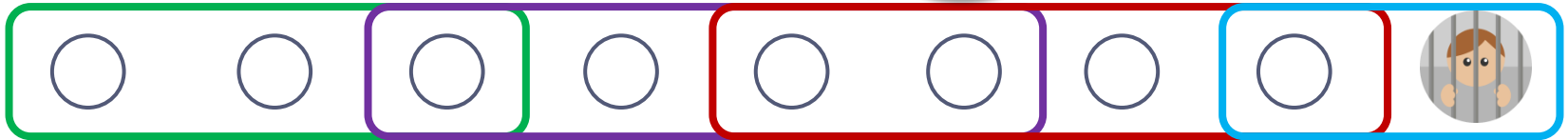


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

COP WINS

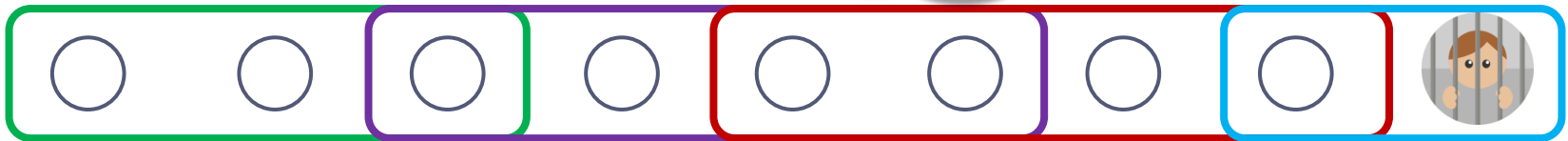


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

COP WINS



Baird

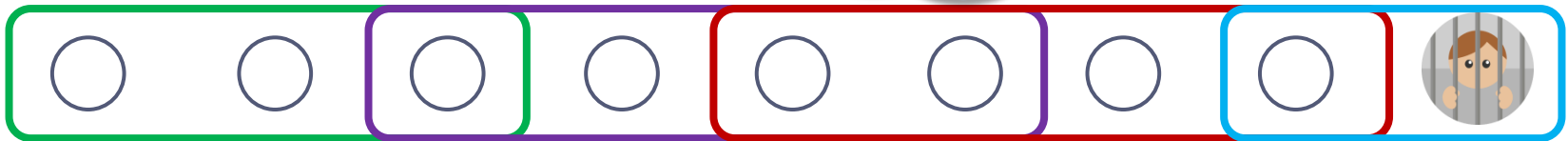
A cycle of length exceed 4 is a robber-win hypergraph

Cop and Robber game played on hypergraphs

Baird

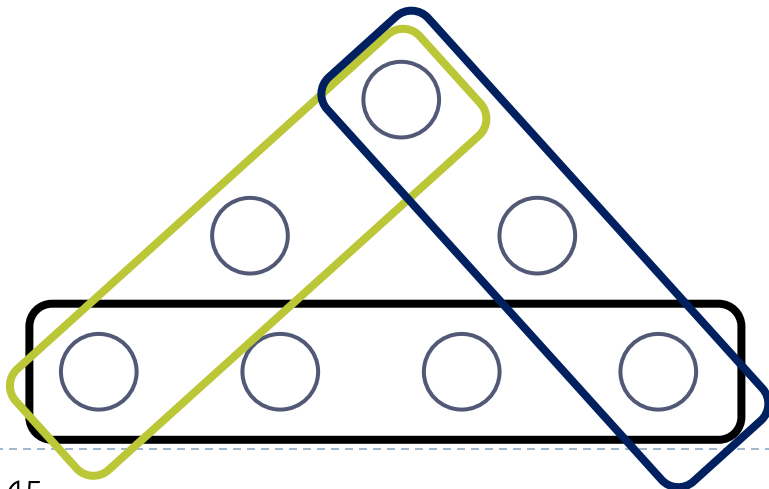
A path is a cop-win hypergraph

COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

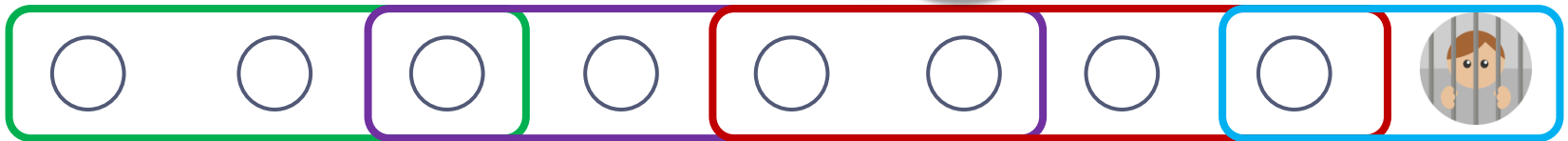


Cop and Robber game played on hypergraphs

Baird

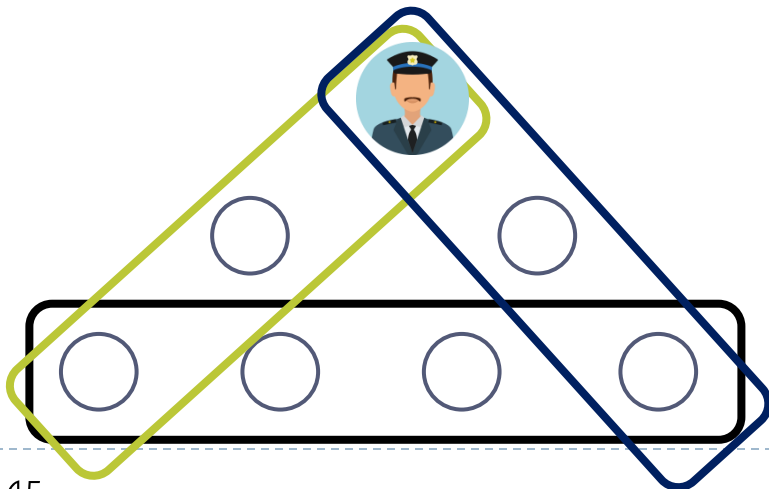
A path is a cop-win hypergraph

COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

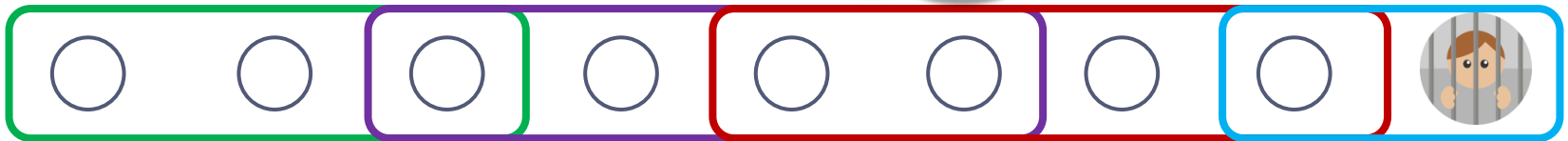


Cop and Robber game played on hypergraphs

Baird

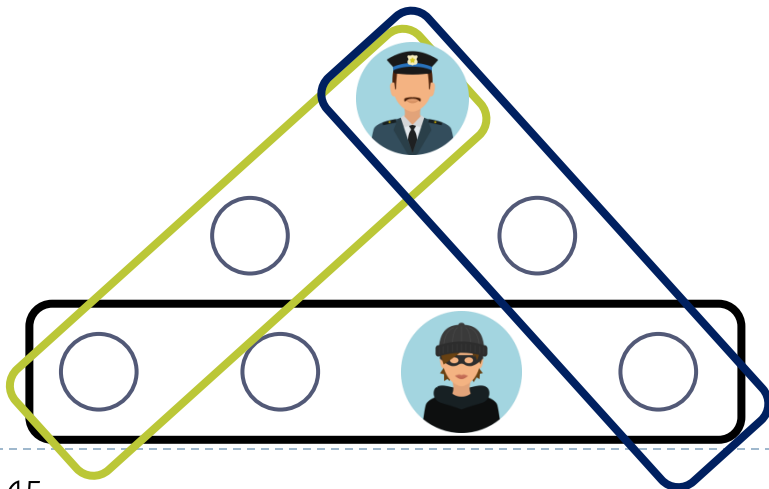
A path is a cop-win hypergraph

COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

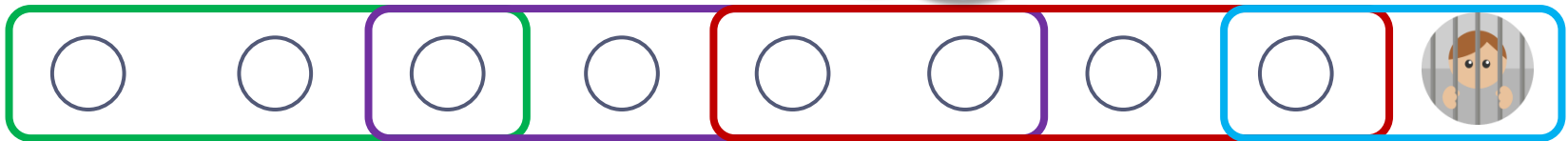


Cop and Robber game played on hypergraphs

Baird

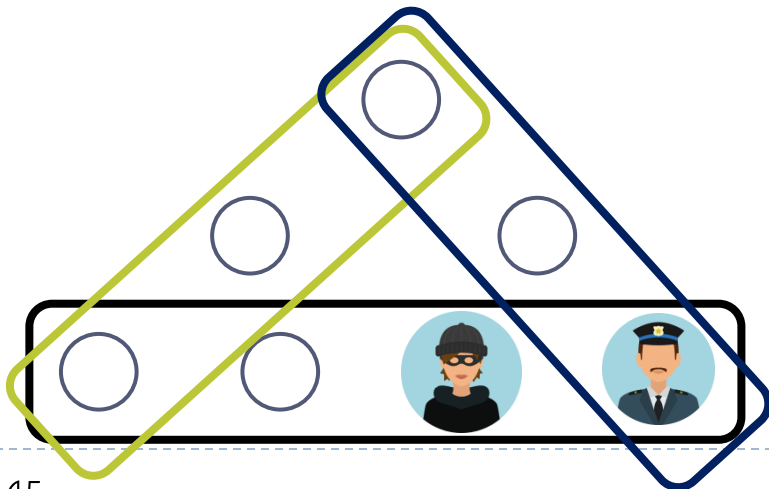
A path is a cop-win hypergraph

COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

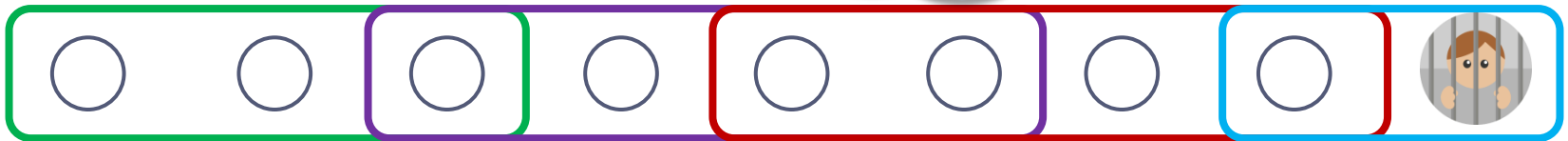


Cop and Robber game played on hypergraphs

Baird

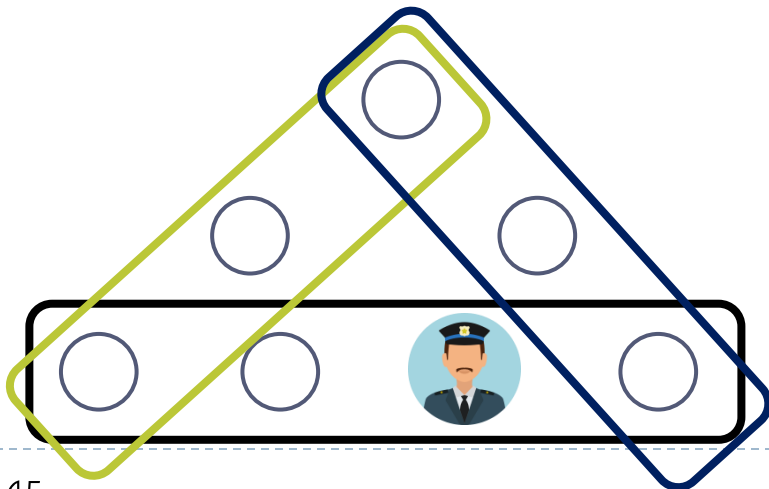
A path is a cop-win hypergraph

COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

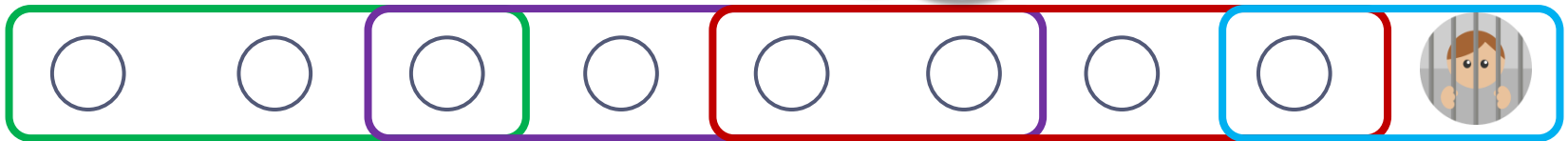


Cop and Robber game played on hypergraphs

Baird

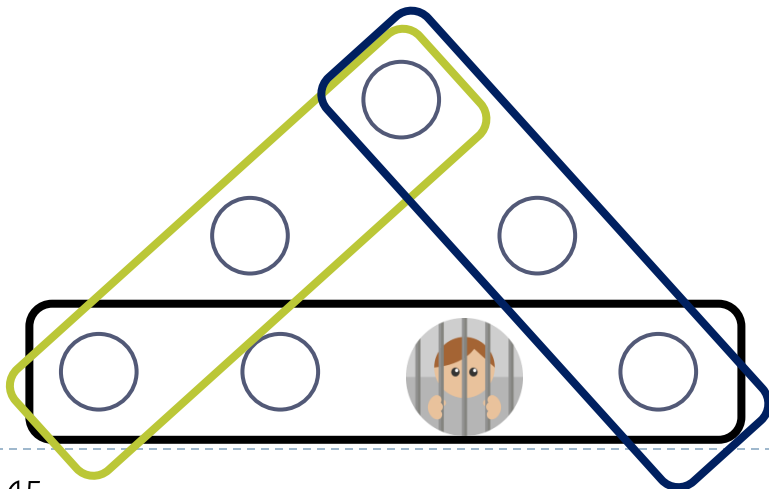
A path is a **cop**-win hypergraph

COP WINS



Baird

A cycle of length exceed 4 is a **robber**-win hypergraph

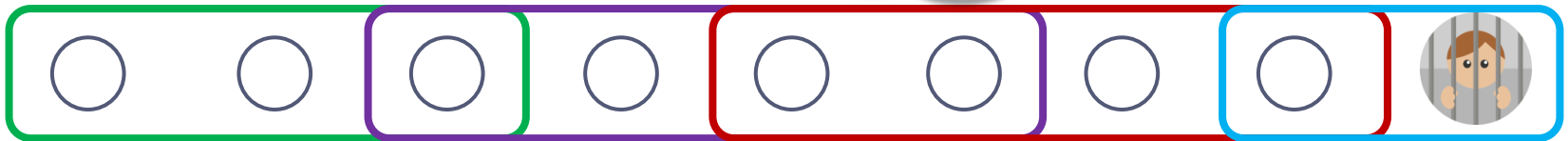


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

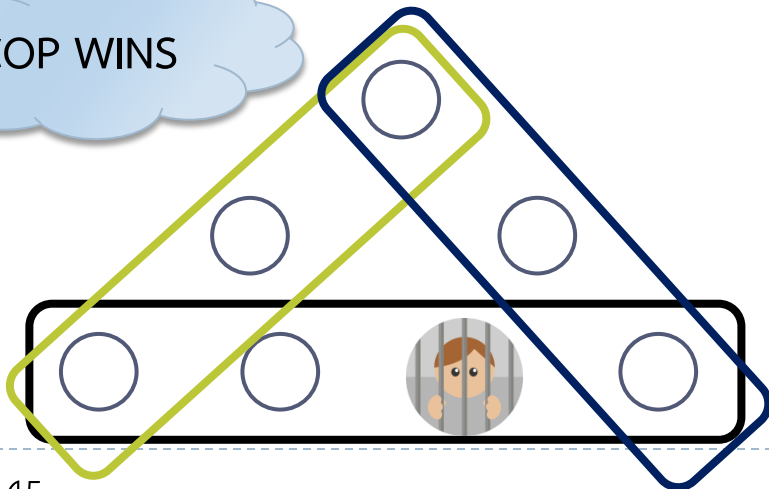
COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

COP WINS

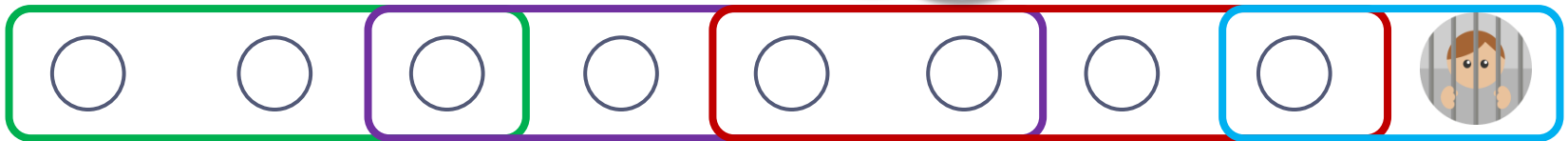


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

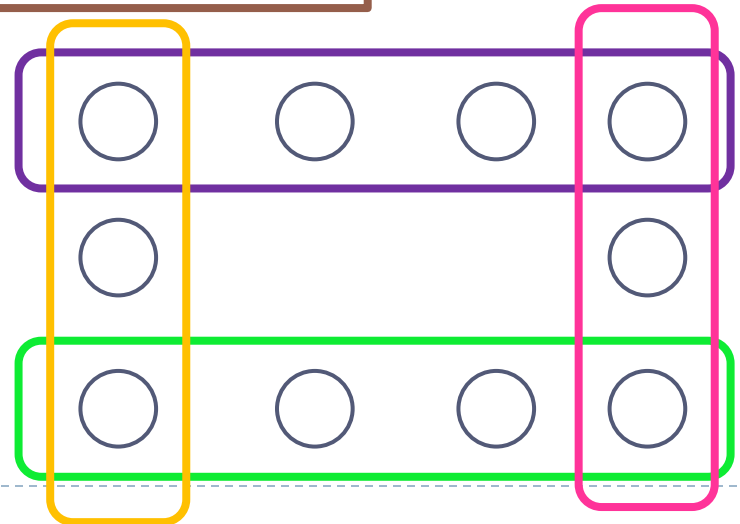
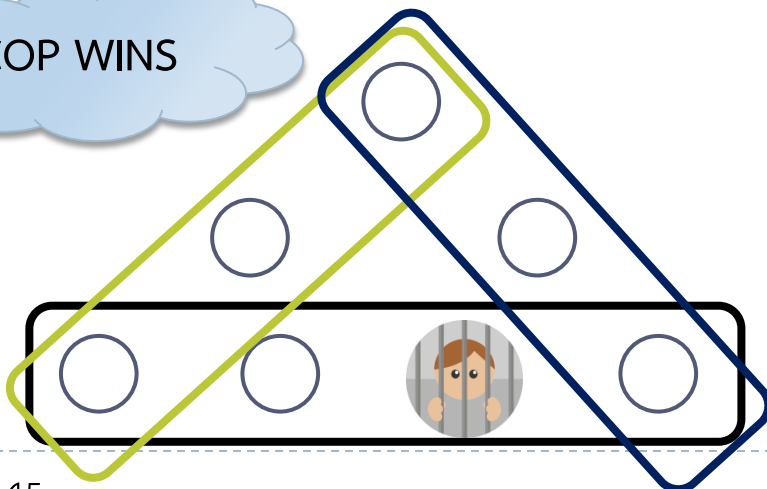
COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

COP WINS

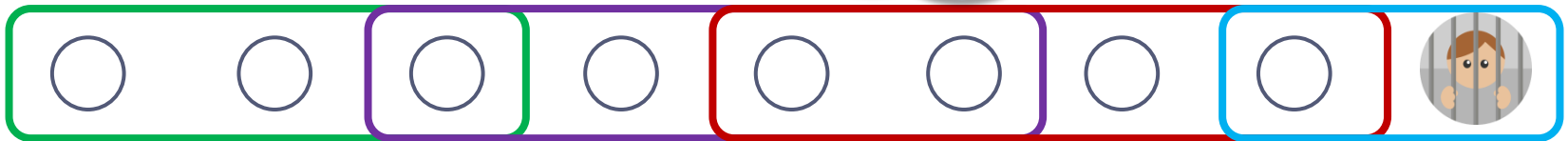


Cop and Robber game played on hypergraphs

Baird

A path is a **cop-win** hypergraph

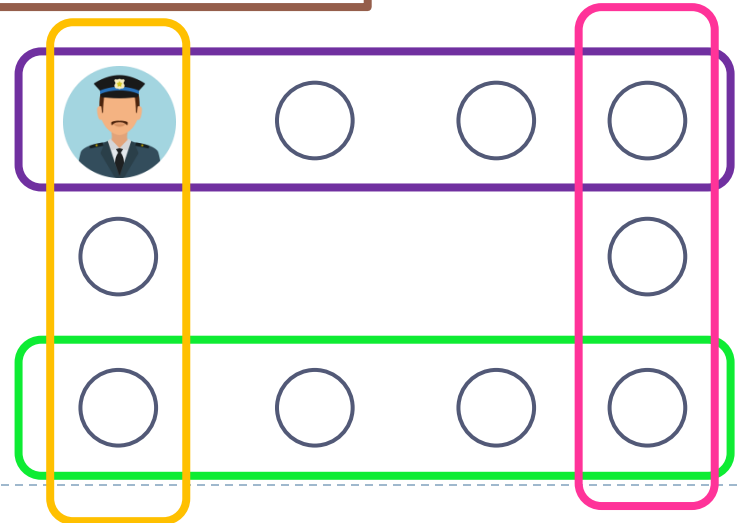
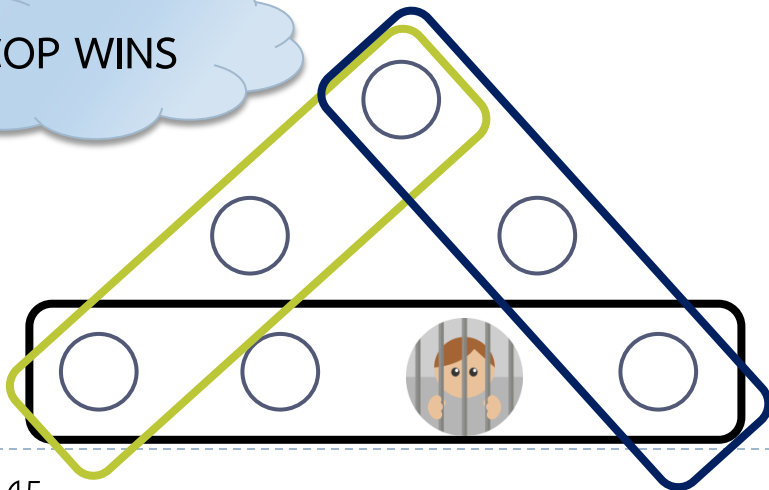
COP WINS



Baird

A cycle of length exceed 4 is a **robber-win** hypergraph

COP WINS

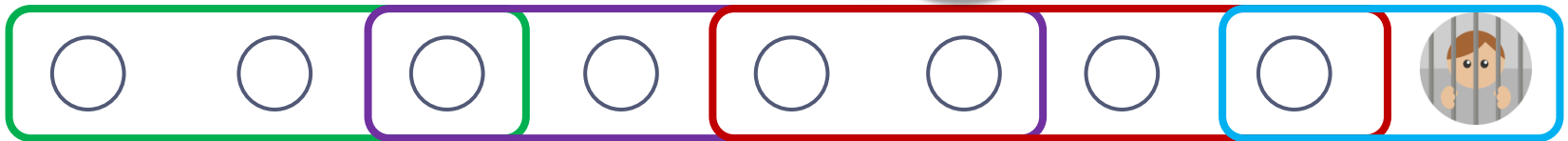


Cop and Robber game played on hypergraphs

Baird

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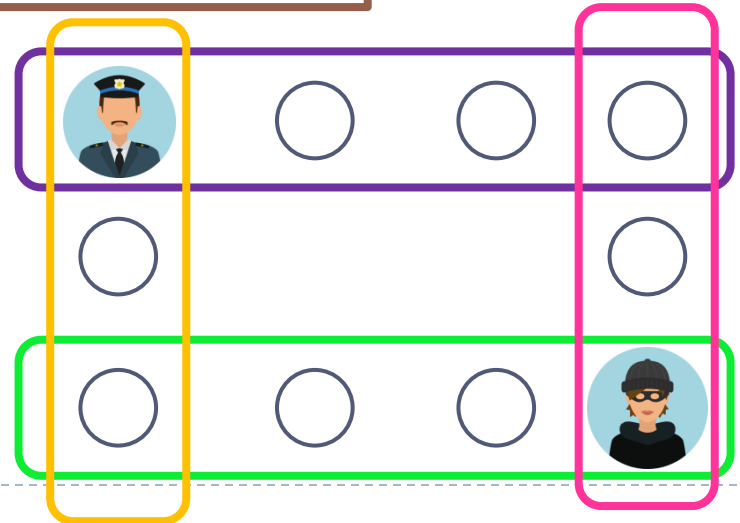
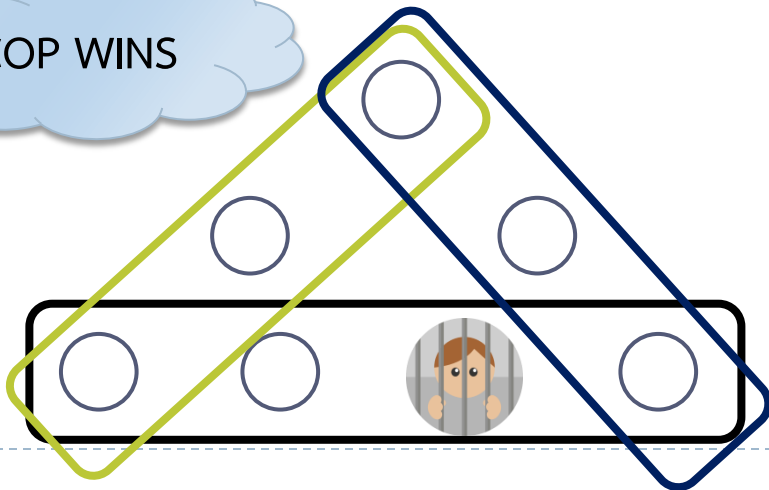
COP WINS



Baird

A cycle of length exceed 4 is a **robber-win** hypergraph

COP WINS

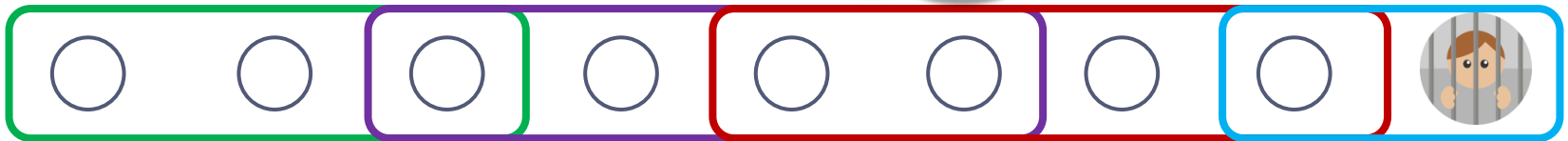


Cop and Robber game played on hypergraphs

Baird

A path is a **cop-win** hypergraph

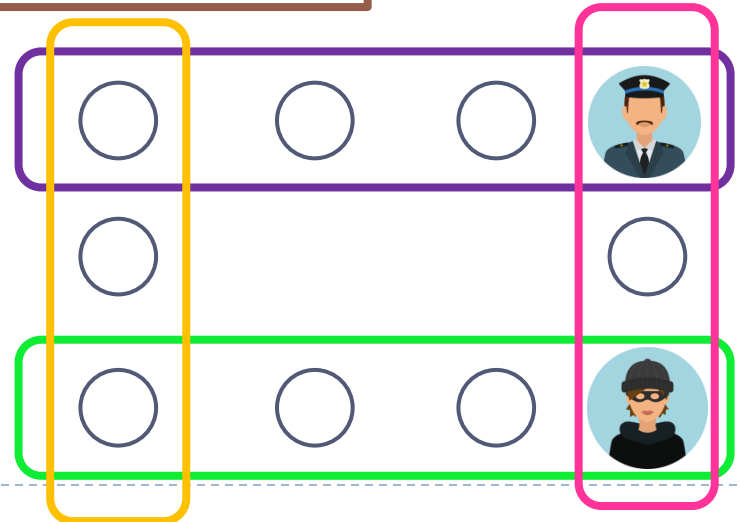
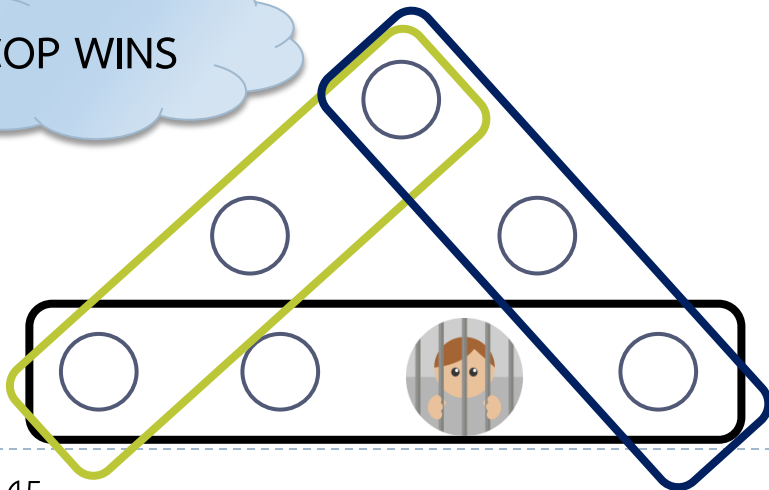
COP WINS



Baird

A cycle of length exceed 4 is a **robber-win** hypergraph

COP WINS

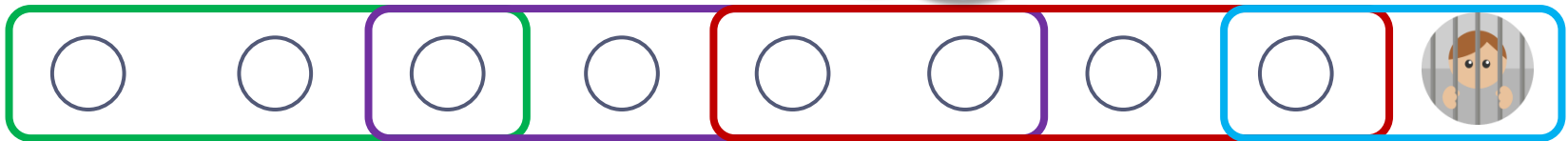


Cop and Robber game played on hypergraphs

Baird

A path is a **cop-win** hypergraph

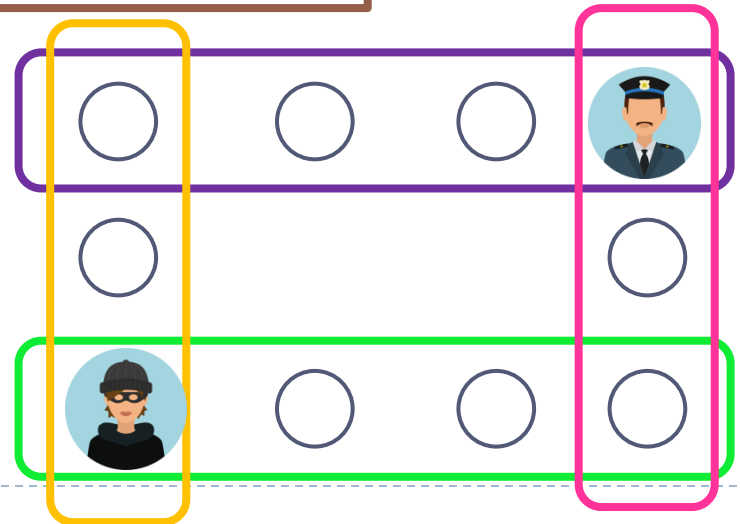
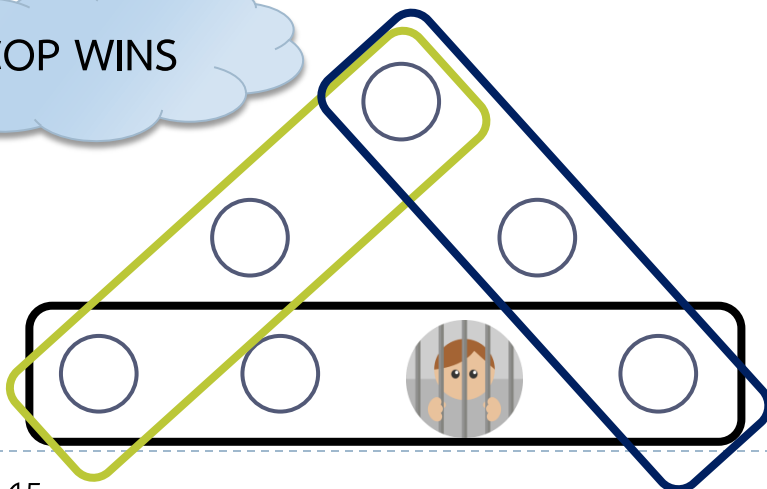
COP WINS



Baird

A cycle of length exceed 4 is a **robber-win** hypergraph

COP WINS

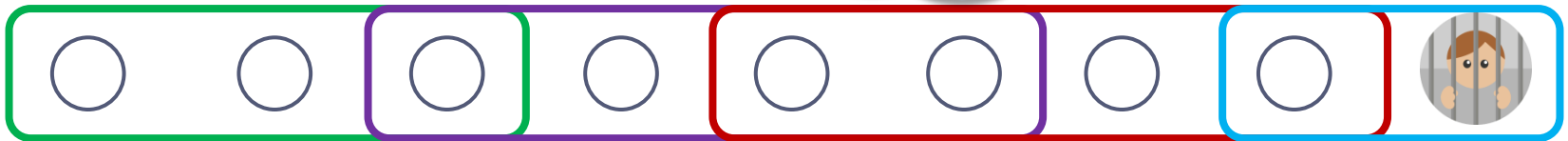


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

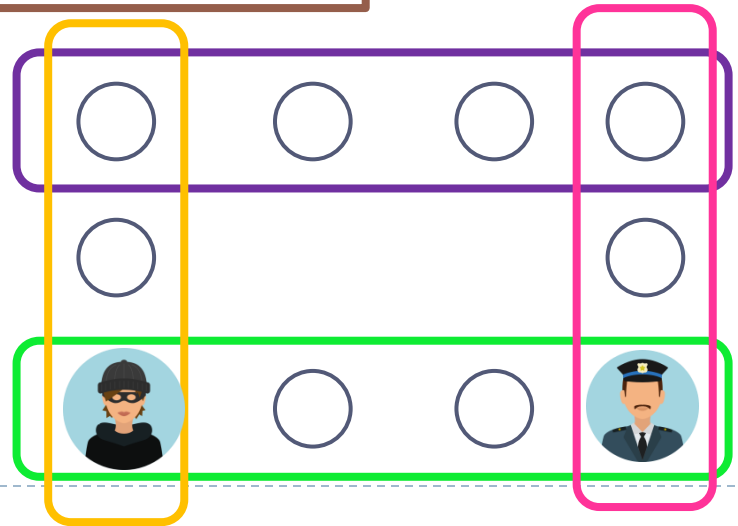
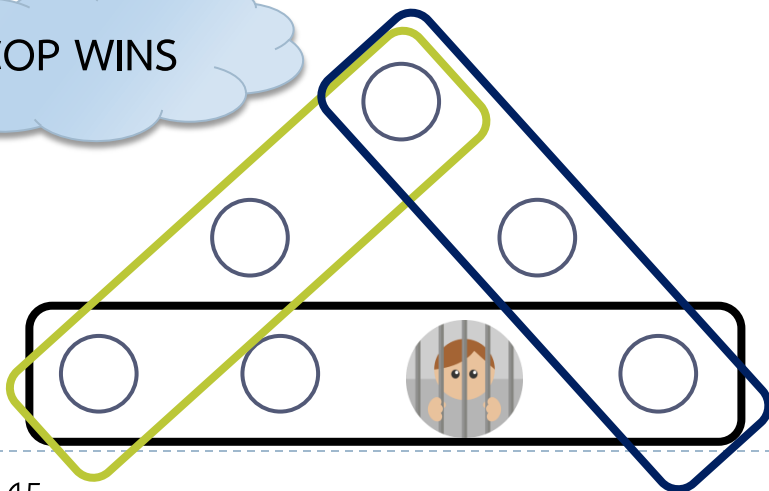
COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

COP WINS

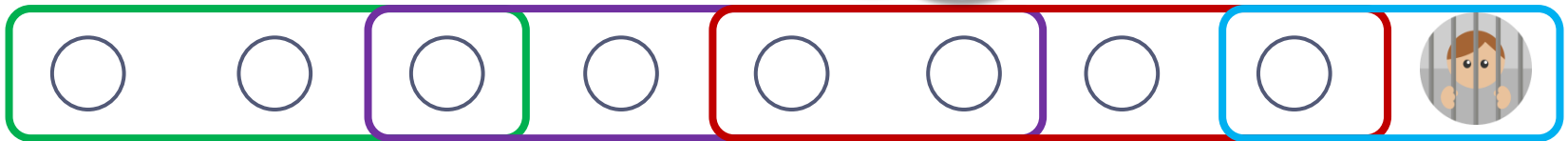


Cop and Robber game played on hypergraphs

Baird

A path is a **cop-win** hypergraph

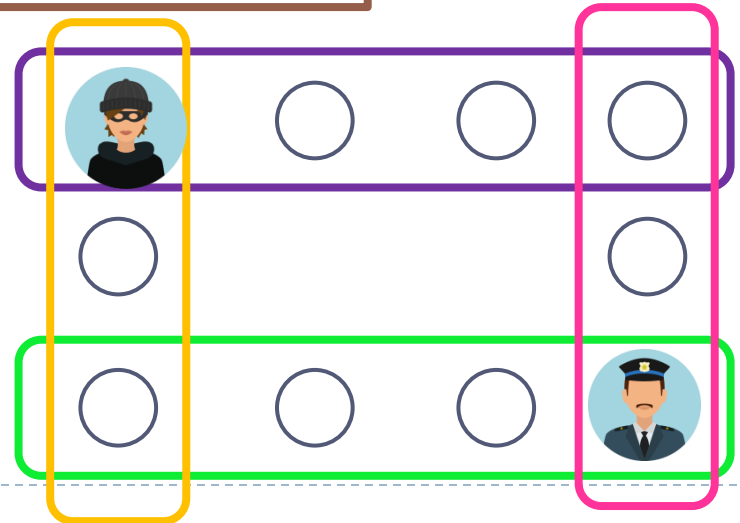
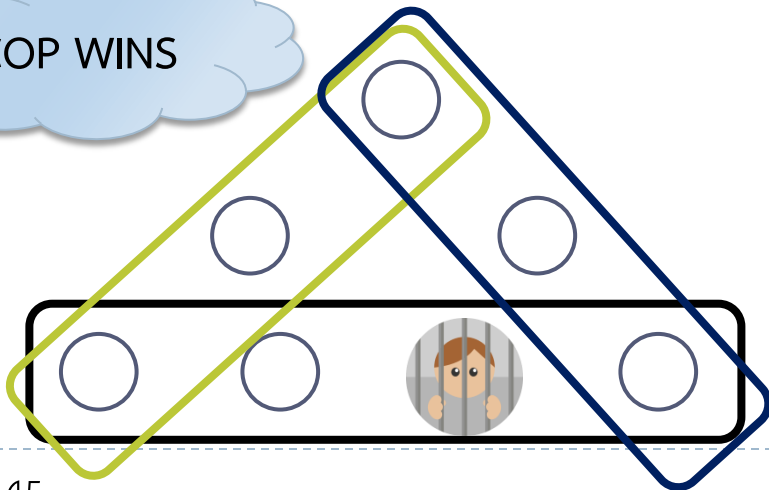
COP WINS



Baird

A cycle of length exceed 4 is a **robber-win** hypergraph

COP WINS

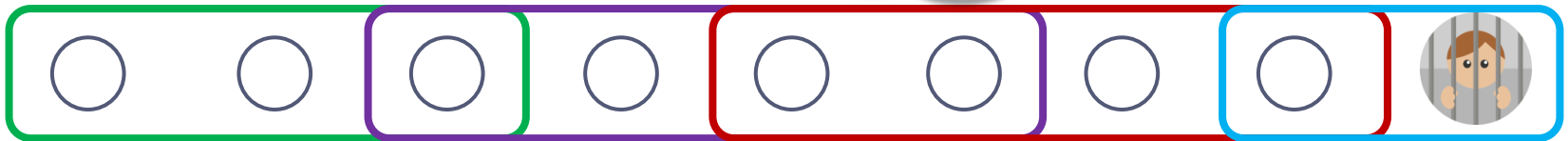


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

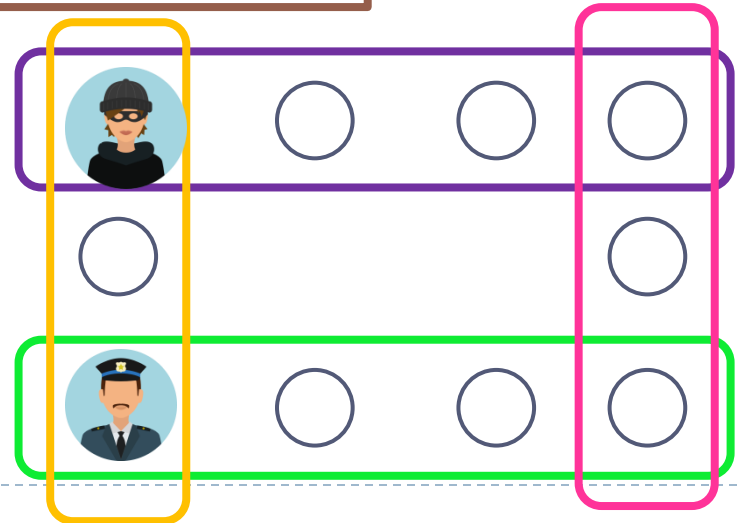
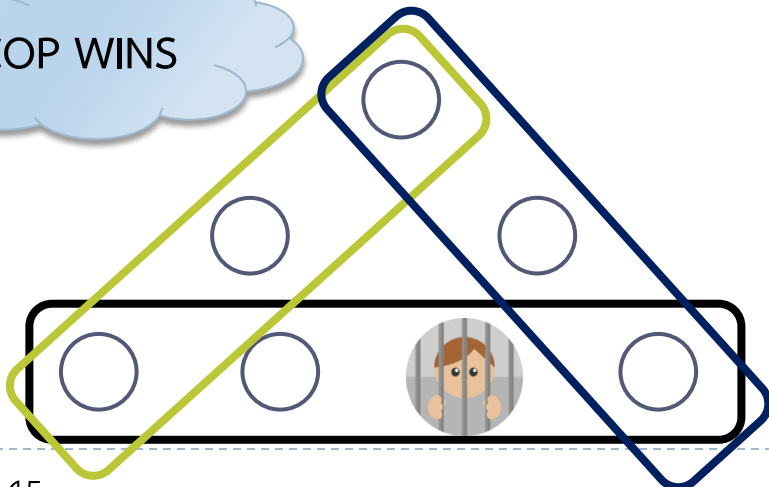
COP WINS



Baird

A cycle of length exceed 4 is a robber-win hypergraph

COP WINS

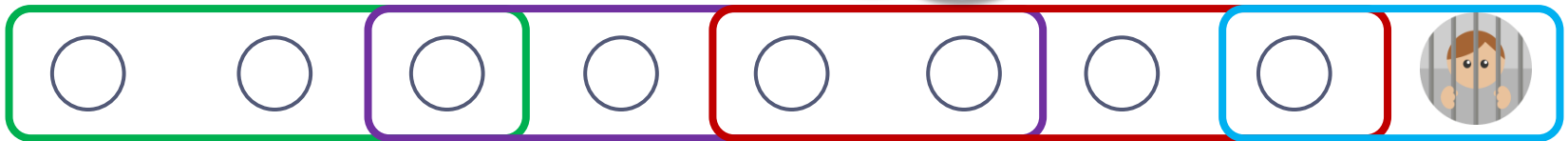


Cop and Robber game played on hypergraphs

Baird

A path is a **cop-win** hypergraph

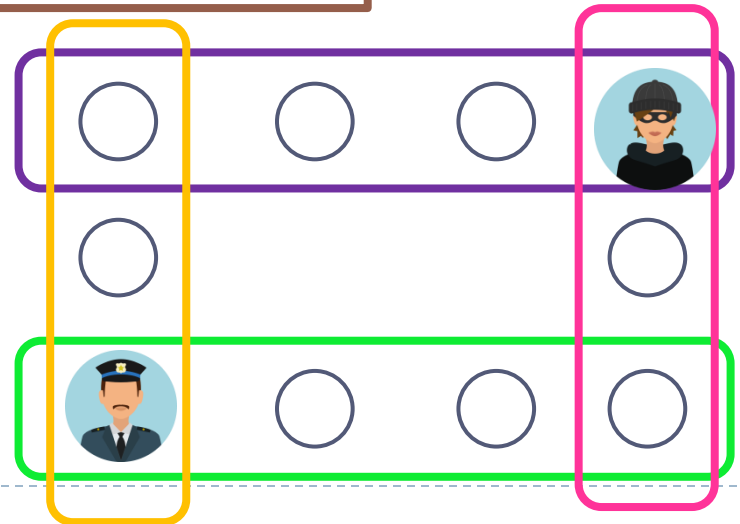
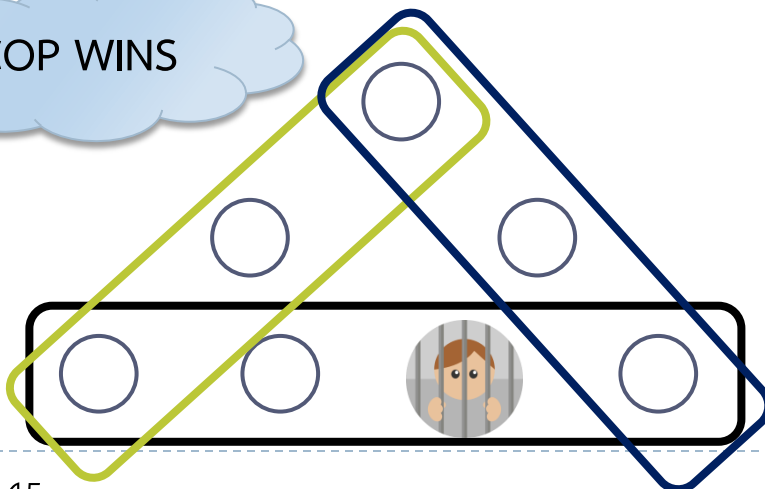
COP WINS



Baird

A cycle of length exceed 4 is a **robber-win** hypergraph

COP WINS

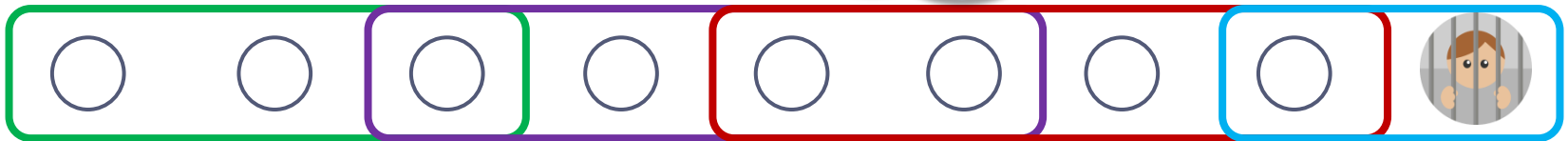


Cop and Robber game played on hypergraphs

Baird

A path is a cop-win hypergraph

COP WINS

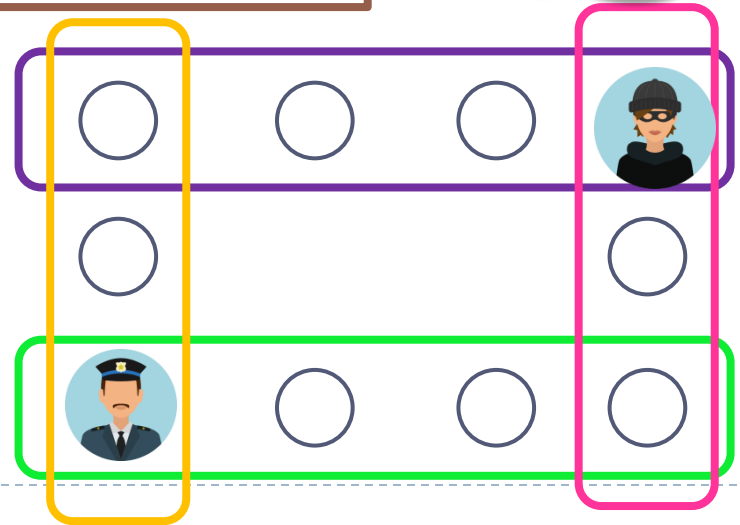
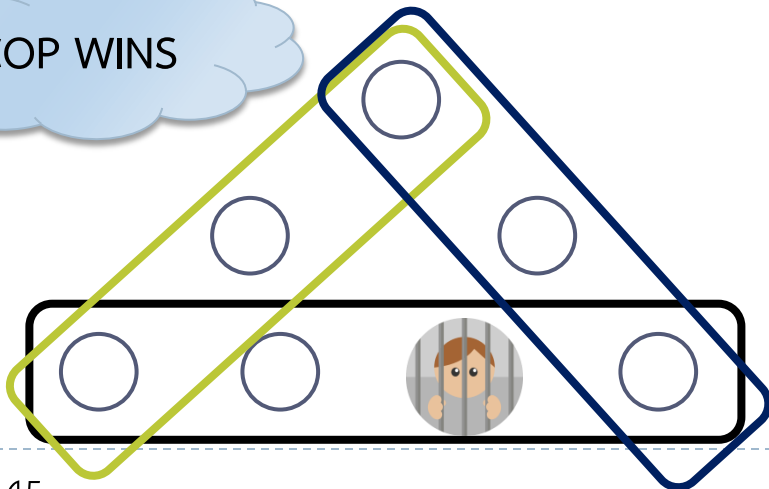


Baird

A cycle of length exceed 4 is a robber-win hypergraph

ROBBER WINS

COP WINS



Cop and Robber game played on hypergraphs



Cop and Robber game played on hypergraphs

Characterize cop-win hypergraphs

Cop and Robber game played on hypergraphs

Characterize cop-win hypergraphs

Generalize some results on graphs to hypergraphs

Cop and Robber game played on hypergraphs

Characterize cop-win hypergraphs

Generalize some results on graphs to hypergraphs

Find the number of cops to catch robber when \mathcal{H} is a robber-win hypergraph

Cop and Robber game played on hypergraphs

Characterize cop-win hypergraphs

Generalize some results on graphs to hypergraphs

Find the number of cops to catch robber when \mathcal{H} is a robber-win hypergraph

Determine the complexity of cop and robber game played on hypergraphs

Catch me! If you can.





